Estimating Nonlinear Investment-q Relation in the Presence of Measurement Error*

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Abstract

We study nonlinearities in the relationship between Tobin's q and firm investment. To capture different elasticities of investment across firms, we augment the classic model of Fazzari, Hubbard, and Petersen (1988) with higher-order terms in q. After correcting for linear and nonlinear measurement errors in q terms, we find evidence of heterogeneity in investment sensitivity. Our estimates reveal that the elasticity of investment is little for firms with low q, and it also starts to decrease for firms with q beyond some intermediate value. In other words, the cross-sectional investment-q relation is first non-decreasing over the support of q, but it then becomes non-increasing for high values of q, resulting in investments clustered around similar levels at both ends of q. This implies that the investment-q relation is not thoroughly linear in the cross section and that a satisfactory increase in investment is unlikely to be achieved by firms with low and high q even if there is a positive change in q.

JEL classification: C21, C26, E22, G31.

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1. Introduction

After Hayashi (1982) formalized that the neoclassical theory of investment originated by Jorgenson (1963) and the q theory by Tobin (1969) are equivalent, many researchers have empirically tested the idea that the optimal rate of investment is solely determined by Tobin's marginal q (i.e., the expected return to investment) using Tobin's average q (i.e., average value of capital) as a proxy for the unobservable true q. While most of the literature after Hayashi (1982) estimate the linear investment equation, only a few studies pay attention to the possibility of a nonlinearity that may reside in the investment-q relation. Although investment is indeed a function of marginal q, the linear functional form has been used for convenience. Moreover, when it comes to the cross-sectional relationship between investment and q, an equation that fits a nonlinear relationship is more needed because firms with different values of q are likely to have different elasticities of investment due to heterogeneous sets of capital to invest (e.g., Eberly, 1997). Our paper fits a nonlinear relationship between investment and q in the cross section by estimating the classic model of Fazzari, Hubbard, and Petersen (1988) augmented with higher-order terms in q. We find that high and low q firms are the least sensitive while intermediate q firms are the most sensitive in adjusting their investment in response to a change of q.

When estimating a polynomial with measurement error like our augmented regression model, the estimation is more complicated because of the multiplicativity of measurement error in the equation. We correct for nonseparable measurement errors in higher-order q terms in polynomials using the instrumental variable (IV) method of Hu and Schennach (2008). The IV estimator enables the identification of nonlinear errors-in-variables models with the aid of an instrument. We adopt an analyst-based measure of average q as the IV, which satisfies sufficient conditions for the identification. Like in the standard IV approach, the conditions include exclusion restrictions and a relevance condition. Also, either monotonicity of investment in the true q or conditional heteroskedasticity is needed, and classical measurement error (i.e., no correlation between measurement error and the true q)

is assumed for the identification. To summarize, since we add higher-order terms in q to the classic regression model to estimate different elasticities of investment across firms, an estimator that can also correct for nonlinear measurement errors is needed. The Hu and Schennach (2008) IV estimator is a solution to the estimation problem.

We find that our augmented model outperforms the classic model in fitting the cross-sectional investment-q relation, as evidenced by significantly higher log-likelihood values. Our bias-corrected augmented model estimated by the IV estimator shows an S-shaped investment-q relation. This indicates that the sensitivity of investment first increases gradually as q improves and then rapidly decreases when q exceeds some intermediate value. This pattern can be observed regardless of what proxy of q is employed. This result implies that it is unlikely that a positive change of q caused by, for example, a corporate tax cut, leads firms with high and low q to significantly increase their investments.

Our paper contributes to the investment literature by revealing how investment sensitivity differs across firms. OLS gives biased results showing that investment and q are not closely related (e.g., Erickson and Whited, 2000). Even after correcting for measurement error, the classic regression model can be still misleading especially for the cross-sectional analysis because it always indicates that firms change their investments at the same rate regardless of their q characteristics. Thus, a biased-corrected augmented regression model is the key to understand an unbiased nonlinear investment-q relation in the cross section.

Our bias-corrected augmented model showing an S-shaped relationship between investment and q predicts that investments of firms with low q are clustered at some low levels. Likewise, investments of firms with high q are predicted to be clustered at some high levels, and in some cases, firms with higher q are predicted to invest even less due to the continuously decreasing response of investment. These results are consistent with theoretical predictions and empirical results in Abel and Eberly (1994), Eberly (1997), Barnett and Sakellaris (1998), Abel and Eberly (2002), and Lee, Shin, and Stulz (2021).

Abel and Eberly (1994) extend the standard theory of investment by adopting an augmented adjustment cost function, which incorporates fixed costs and irreversibility of investment. Investment is a nondecreasing function of marginal q and is in one of three regimes (positive, zero, or negative gross investment) in their model. The regime of zero gross investment makes the response of investment to q nonlinear. Abel and Eberly (2002) and Barnett and Sakellaris (1998) empirically test this idea and find a nonlinear S-shaped relationship between investment and q with no regime of zero gross investment. Eberly (1997) also find a nonlinear but convex relationship between investment and q from international data. In a recent study, Hoberg and Maksimovic (2021) develop a four-stage 10-K text-based model of product life cycles, which includes q interacted with proxies for firm-year product life cycle exposures. They find that firms initially focus on research and development (R&D). Capital expenditures (CAPX), acquisitions, and divestitures then emerge in order as firms mature in their life cycles. These papers estimate how the elasticity of investment develops as q changes within firm. On the other hand, our paper estimates the cross-sectional variation in investment sensitivity. In another recent paper, Lee, Shin, and Stulz (2021) show that investment does not increase in the cross section with q for large firms and that the sensitivity of investment to q falls as firms become older and larger. They instead find that repurchases of large and old firms increase with q in the cross section, arguing that q is more of a proxy for rents from the past investment than investment opportunities for such firms. We also find that net payout consistently increases with q, which can be one of reasons for the decreasing elasticity of investment.

One early strand of the literature examined the empirical failure of the neoclassical theory of investment. The empirical formulation of the theory performs poorly with real data compared to augmented models with proxies of financial constraints such as cash flow. Fazzari, Hubbard, and Petersen (1988) find that when firms are financially constrained, investment spending also varies with the availability of internal funds. This finding may reflect the existence of asymmetric information in financial markets and thus contradict the assumption of perfect capital markets in the neoclassical theory. Fazzari, Hubbard, and Petersen (1988) and the subsequent literature (e.g., Gilchrist and Himmelberg, 1995; Kaplan and Zingales, 1997; Cleary, 1999) are questioned by Erickson and Whited (2000), arguing that the neoclassical model cannot perform well if marginal q is mismeasured. Since marginal q is unobservable, average q is used instead. This is where a measurement error problem arises since marginal q and average q are different in the real world.[1] Using a measurement-error-robust generalized method of moments estimator on balanced panel data, they find that cash flow is not relevant to investment decisions even for financially constrained firms, which corroborates good predictive power of the q theory in the absence of measurement error. Cummins, Hassett, and Oliner (2006) also cast doubt on the appropriateness of the existing measure of q, which uses stock prices to proxy for intrinsic value, pointing out its uninformativeness and thus persistent measurement error in it. Using financial analysts' earnings forecasts instead, they construct an analyst-based measure of q and find that investment is not sensitive to cash flow. Although positive coefficients on cash flow are observed in a later work by Erickson and Whited (2012) using the high-order moment estimator on unbalanced panel data, they emphasize that those estimates are much smaller in magnitude and significance than the ordinary least squares (OLS) counterparts. Erickson, Jiang, and Whited (2014) then apply a new closed-form cumulant estimator to the investment equation and find results consistent with Erickson and Whited (2000, 2012).

On the other hand, Almeida, Campello, and Weisbach (2004) find that constrained firms save cash out of cash flows, which suggests that the effect of financial constraints on corporate policies is manifested in the form of firms' demand for liquidity. Almeida and Campello (2007) also find that investment-cash flow sensitivity increases in the tangibility of a firm's assets when the firm is financially constrained, which indicates that the influence of financing frictions on investment decisions is multiplied by asset tangibility. Almeida, Campello, and

[1] Assuming that the profit function and the adjustment cost function are linearly homogeneous, Hayashi (1982) and Abel and Eberly (1994) show that the firm value is equal to the marginal q times the capital stock, and thus average q and marginal q are essentially the same. It is well known that if firm is a price taker in output and factor markets with constant returns to scale in production, the profit function is linearly homogeneous.

Galvao (2010) assess the performance of methods dealing with measurement error in q and find that investment decisions are consistently affected by cash flow. Lewellen and Lewellen (2016) find results that cash flow still matters for investment decisions after considering its correlation with q, and Ağca and Mozumdar (2017) also show that cash flow is a significant predictor of investment after employing different estimation methods and alternative proxies of q. Firpo, Galvao, and Song (2017) estimate the investment equation using linear quantile regression and find that the investment-cash flow sensitivity is relatively stronger at the lower part of the conditional distribution of investment. With their own efforts to resolve the measurement error problem, these studies provide evidence that internal funds consistently matter for investment decisions. The interpretation of an investment-cash flow sensitivity is, however, controversial. Even in the absence of financing constraints, investment is predicted to be responsive to the availability of internal funds in recent theories (e.g., Abel and Eberly, 2011; Gourio and Rudanko, 2014; Abel, 2018).

The investment literature has always been an actively studied area, and there are important recent studies in corporate finance that contribute to the literature. Peters and Taylor (2017) find that q explains investment better when intangible capital is taken into account. They show that the so-called "total q" is superior to the standard measure of q in that it can proxy for both physical and intangible investment opportunities. Woeppel (2021) introduces another new measure of Tobin's q. Patent q that incorporates the replacement cost of patent capital is shown to strengthen the historically weak investment-q relation. Andrei, Mann, and Moyen (2019) find that the investment-q relation itself has also become tighter in recent years than earlier times and that the reason is on the growing empirical dispersion in q both in the cross-section and the time series.

This paper proceeds as follows. Section 2 describes the data. Section 3 describes the identification strategy for addressing measurement errors in augmented models. Section 4 presents the results. Section 5 discusses the results. Section 6 concludes.

2. Data

We use data from 1982 to 2017. Our sample includes all manufacturing firms on Compustat except utilities (Standard Industrial Classification codes 4900 – 4999), financial firms (6000 – 6999), and public administration firms (9000 – 9999). We require firms to be covered by IBES to construct our instrumental variable q^{IV}. We further discuss our instrument later in this section. We deflate all series to 1990 dollars using the Consumer Price Index. We exclude firm-year observations with missing or non-positive book value of assets or sales and with missing or less than \$5 million real 1990 dollars in gross property, plant, and equipment (PP&E). We also exclude firm-years with negative q^{IV}, which implies a negative firm value, or with q^{IV} in excess of 50, which is an unrealistically large value.[2]

Many studies in the literature use balanced panels with relatively short sample periods (e.g., Gilchrist and Himmelberg, 1995; Himmelberg and Petersen, 1994; Whited, 1992). We instead use an unbalanced panel, so survivorship bias is not our concern.[3] We drop observations with missing value of our regression variables. To remove extreme outliers, we winsorize our regression variables over the entire panel at once at the 1st and 99th percentiles. Our final sample consists of 58,796 firm-years. The first year is 1983 because the specification includes lagged values.

2.1. Tobin's q, investment, and cash flow measures

Peters and Taylor (2017) show that their new measure of Tobin's q, which is total q, explains both physical and intangible investments better by accounting for intangible capital. Thus, we consider both the physical measure of Tobin's q and total q in this paper.

[2]For example, Almeida and Campello (2007) and Cummins, Hassett, and Oliner (2006) delete firm-years with negative average q. Researchers sometimes discard observations with unrealistically large average q as a crude attempt to limit the impact of measurement error. For example, Almeida and Campello (2007) and Gilchrist and Himmelberg (1995) eliminate observations with average q exceeding 10. Abel and Eberly (2002) restrict average q to be less than 5. Eberly (1997) drops observations with average q in excess of 15.

[3]Almeida and Campello (2007) and Barnett and Sakellaris (1998) also use unbalanced panels, requiring at least three years and five consecutive years of data for each firm, respectively. The minimum number of years required for firms in Almeida and Campello is based on the lag structure of the regression models and their IV approach.

In the literature, physical q is measured by scaling firm value by the replacement cost of physical assets:

$$q_{it}^{phy} = \frac{V_{it}}{K_{it}^{phy}}. \tag{1}$$

We measure firm value V as the market value of equity (Compustat items *prcc_f*$\times$*csho*) plus the book value of debt (Compustat items *dltt*+*dlc*) minus the book value of current assets (Compustat item *act*). The replacement cost of physical capital, denoted by K^{phy}, is measured as the book value of gross PP&E (Compustat item *ppegt*).

Total q, denoted by q^{tot}, is measured by scaling firm value by the replacement cost of total capital, denoted by K^{tot}, which is the sum of physical and intangible capital:

$$q_{it}^{tot} = \frac{V_{it}}{K_{it}^{tot}} = \frac{V_{it}}{K_{it}^{phy} + K_{it}^{int}}. \tag{2}$$

The replacement cost of intangible capital, denoted by K^{int}, is downloaded from Peters and Taylor Total Q (Variable *K_int*) on Wharton Research Data Services.

Physical investment, denoted by i^{phy}, is measured as capital expenditures (Compustat item *capx*) divided by the replacement cost of lagged physical capital:

$$i_{it}^{phy} = \frac{CAPX_{it}}{K_{i,t-1}^{phy}}. \tag{3}$$

Total investment, denoted by i^{tot}, is measured as the sum of capital expenditures and intangible investment (i.e., R&D (Compustat item *xrd*) plus 30% of Selling, General and Administrative (SG&A) expenses (Compustat items $xsga - xrd - rdip$)) divided by the replacement cost of lagged total capital[4]:

$$i_{it}^{tot} = \frac{CAPX_{it} + R\&D_{it} + 0.3 \times SG\&A}{K_{i,t-1}^{tot}}. \tag{4}$$

[4]We subtract R&D from *xsga* to isolate non-R&D SG&A when Compustat adds R&D to *xsga*, following Peters and Taylor (2017).

Cash flow, denoted by c, is measured as income before extraordinary items (Compustat item ib) plus depreciation (Compustat item dp) divided by lagged physical capital. We also construct Peters and Taylor's (2017) measure of cash flow c^{tot}, which is called total cash flow, by adding tax-adjusted intangible investment back into the free cash flow in the numerator and dividing it by lagged total capital:

$$c_{it} = \frac{ib_{it} + dp_{it}}{K^{phy}_{i,t-1}}, \quad c^{tot}_{it} = \frac{ib_{it} + dp_{it} + (CAPX_{it} + R\&D_{it} + 0.3 \times SG\&A)(1-\tau)}{K^{phy}_{i,t-1} + K^{int}_{i,t-1}}, \tag{5}$$

where τ is the marginal tax rate. We use the simulated marginal tax rate based on income before interest expense from Graham (1996) when available. Otherwise, a marginal tax rate is assumed to be 30%, which is close to the average marginal tax rate in our sample.

2.2. *Analyst-based measure of Tobin's q*

Our identification strategy relies on the availability of an instrumental variable. We construct an analyst-based measure of average q, analyst q, following the idea of Cummins, Hassett, and Oliner (2006) and use it as our instrument. Compared to the existing measures of q, analyst q is only different in the numerator in which equity value is measured based on analysts' earnings forecasts on IBES.

We focus on the means (IBES item *meanest*) of analysts' forecasts of earnings per share (IBES item *measure*=EPS) over the current (IBES items *fiscalp*=ANN with *fpi*=1) and the next fiscal years (IBES items *fiscalp*=ANN with *fpi*=2) and long-term growth forecasts (IBES items *fiscalp*=LTG with *fpi*=0), which in general represent the analysts' consensus on forecast of the average annual growth of earnings over the three years after the next fiscal year. In the neoclassical theory of investment, firms make investment decisions based on the expected returns to capital in the long-term. Thus, a long-term growth forecast could be an important determinant of investment. Due to the limited availability of long-term growth forecasts in IBES, our sample starts in 1982.

Shortly after the beginning of a firm's fiscal year, analysts send IBES initial forecasts of earnings for that year and the next few fiscal years. We use the first forecasts for the current and the next fiscal years as well as the first long-term growth forecast to reduce the risk of using more information than the firm actually has when deciding its investment spending for the current fiscal year. We then measure the analyst-based measure of equity value $\hat{E}$ using those forecasts and construct $\hat{q}$, an analyst-based measure of q, as described in Cummins, Hassett, and Oliner (2006):

$$\hat{q}^{phy}_{i,t-1} = \frac{\hat{E}_{it} + dltt_{i,t-1} + dlc_{i,t-1} - act_{i,t-1}}{K^{phy}_{i,t-1}}, \quad \hat{q}^{tot}_{i,t-1} = \frac{\hat{E}_{it} + dltt_{i,t-1} + dlc_{i,t-1} - act_{i,t-1}}{K^{tot}_{i,t-1}}, \quad (6)$$

where $\hat{E}_{it} = \Pi_{it} + \frac{1}{1+r_t}\Pi_{i,t+1} + \frac{1}{(1+r_t)^2}\overline{\Pi}_{it}(1+\mathrm{LTG}_{it}) + \frac{1}{(1+r_t)^3}\overline{\Pi}_{it}(1+\mathrm{LTG}_{it})^2 + \frac{1}{(1+r_t)^4}\overline{\Pi}_{it}(1+\mathrm{LTG}_{it})^3 + \frac{1}{(1+r_t)^4}\frac{1}{\bar{r}-\bar{g}}\overline{\Pi}_{it}(1+\mathrm{LTG}_{it})^4$. Π_{it} and $\Pi_{i,t+1}$ represent the averages of earnings forecasts per share over the current and the next fiscal years times the corresponding number of shares outstanding (IBES item *shout*) of each fiscal year. $\overline{\Pi}_{it}$ is the average of the two annual forecasts. LTG_{it} is the mean of analysts' long-term growth forecasts. We use the one-year Treasury bill rate in year t plus an assumed equity risk premium of 8% as a discount factor r_t, which reflects the annual nominal equity return expected by investors in year t. $\bar{r}$ is the mean nominal equity return (12%) during the sample period. $\bar{g}$ is the mean growth rate of nominal GDP (5%) during the sample period. Other items are lagged by one year to construct lagged q and to match the time when $\hat{E}_{it}$ becomes available.

Analyst q can serve as a valid instrument for the identification of the true investment-q relation. Considering the forward-looking nature of earnings forecasts and at the same time the independence of data source owned by well-informed professional experts, the analyst-based measure of equity value is not identical to the market value of equity while highly correlated with it (Abel and Eberly, 2002). As a result, analyst q is correlated with the corresponding market-based q while unlikely to be correlated with measurement error in it.

2.3. Summary statistics

Table 1 contains summary statistics. Total investment shows a different empirical distribution, which is slightly right to the distribution of physical investment. Considering that total investment is scaled by a larger denominator K^{tot} than physical investment is, the amount of intangible investment itself is expected to be large. Our consideration of intangible capital and intangible investment can thus help explore the true relationship between investment and q in the cross section.

[Table 1 here]

The analyst-based measures of q are consistently higher than the corresponding market-based measures of q in every statistic describing the distributions. This reflects the observation that analysts' forecasts are overoptimistic (e.g., De Bondt and Thaler, 1990; Easterwood and Nutt, 1999). In other words, analysts tend to believe that firms have better growth opportunities than the financial markets perceive. Nevertheless, both types of q are highly correlated. The correlation between q and $\hat{q}$ is 0.79, and the correlation between q^{tot} and $\hat{q}^{tot}$ is 0.75.

We also construct other versions of total q. Using Woeppel's (2021) estimate of intangible capital (i.e., patent capital plus on-balance sheet intangible capital), we construct patent q. EPW q is constructed based on Ewens, Peters, and Wang's (2020) estimate of intangible capital. Patent q is overall higher than total q and EPW q because patents are only considered as internally-generated intangible capital in patent q, so its denominator is smaller than the other two proxies'. Nevertheless, patent q still gives a more reasonable range of q than physical q.

Total cash flow has overall lower values than standard cash flow, and this is due to the larger denominator K^{tot}, although tax-adjusted intangible investment is added back in the numerator. Net payout is calculated as cash dividends plus the purchase of common and preferred stock minus the sale of common and preferred stock, scaled by lagged total capital.

If the calculation yields a negative or missing value, net payout is set to zero.

3. Identification strategy and implementation

The investment literature has been interested in the true investment-q relation, and most studies estimate a linear relationship in various ways such as the higher-order moment estimator (e.g., Erickson and Whited, 2000) and instrumental variable approaches (e.g., Almeida, Campello, and Weisbach, 2004; Almeida and Campello, 2007). Considering potential nonlinearities in the investment-q relation, we fit nonlinear relationship to the data to detect different investment sensitivities across firms. Polynomials with mismeasured regressors such as the classic model of Fazzari, Hubbard, and Petersen (1988) with higher-order terms in q inevitably contain errors nonseparable from the true q. Such nonlinear errors-in-variables models can be estimated through an eigenvalue-eigenfunction decomposition following Hu and Schennach (2008).

The true investment equation is defined by the joint distribution of investment and marginal q. Since marginal q is unobservable, the error-contaminated counterpart, average q is used instead in empirical research. According to Hu and Schennach (2008), the joint distribution of investment and marginal q is identifiable from the distribution of all observed variables including an instrumental variable. Also, their treatment of measurement error models can be extended to allow for the presence of a vector of additional correctly measured regressors by conditioning all densities on that vector (Song, 2015). Cash flow can thus be included in the estimation, which is essential considering its explanatory power regarding investment.

We need investment (i), average q (q^A), cash flow (c), and an instrument (q^{IV}) for the identification of the joint distribution of investment, marginal q (q^M), and cash flow. We are interested in the true investment model expressed in terms of the observed i and c and the unobserved q^M:

$$f_{i|q^M c}(i|q^M, c). \tag{7}$$

As long as we correctly identify the conditional density of investment conditional on marginal q and cash flow, we can estimate the true investment equation. Since marginal q is unobserved, we identify the conditional density using the innovative identification strategy from the literature on nonlinear errors-in-variables models.

Let $\mathcal{I}$, $\mathcal{Q^A}$, $\mathcal{Q^M}$, $\mathcal{C}$, and $\mathcal{Q^{IV}}$ denote the supports of the distributions of the random variables i, q^A, q^M, c, and q^{IV}. Assumptions 1–4 in the Appendix lead to the theorem that enables the identification of unknown densities. Theorem 1 in the Appendix states that under Assumptions 1–4, given the observed density $f_{i\,q^A|c\,q^{IV}}(i, q^A|\, c, q^{IV})$, the equation

$$f_{i\,q^A|c\,q^{IV}}(i, q^A|\, c, q^{IV}) = \int_{\mathcal{Q^M}} f_{i|q^M c}(i|q^M, c) f_{q^A|q^M c}(q^A|q^M, c) f_{q^M|q^{IV} c}(q^M|q^{IV}, c)\, dq^M \tag{8}$$

admits a unique solution $(f_{i|q^M c}, f_{q^A|q^M c}, f_{q^M|q^{IV} c})$ for all $i \in \mathcal{I}$, $q^A \in \mathcal{Q^A}$, $c \in \mathcal{C}$, $q^{IV} \in \mathcal{Q^{IV}}$. This theorem corresponds to the identification results in Hu and Schennach (2008) and Song (2015). These papers ensure that models satisfying Assumptions 1–4 can be easily constructed because they are not mutually contradictory.

Given the true model in Equation (7), Theorem 1 suggests the following measurement-error-robust maximum likelihood estimator:

$$(\hat{\theta}_0, \hat{\theta}_1, \hat{\theta}_2) =$$
$$\arg\max_{(\theta_0,\theta_1,\theta_2)\in\Theta} \frac{1}{n}\sum_{j=1}^{n} \ln \int_{\mathcal{Q^M}} f_{i|q^M c}(i_j|q^M, c_j; \theta_0) f_{q^A|q^M c}(q^A_j|q^M, c_j; \theta_1) f_{q^M|q^{IV} c}(q^M|q^{IV}_j, c_j; \theta_2)\, dq^M, \tag{9}$$

where Θ is the parameter space. The analyst-based measure of q is used as the instrument q^{IV}. When the dependent variable is i^{phy}, q^{phy}, $\hat{q}^{phy}$, and standard cash flow c are employed. Similarly, when the dependent variable is i^{tot}, q^{tot}, $\hat{q}^{tot}$, and c^{tot} are the regressors.

The IV estimator covers from parametric estimation of a linear model with classical measurement error to nonparametric estimation in the presence of nonclassical measurement error. We approximate the unobservable conditional densities by normal distribution with the assumptions of classical measurement error and conditional heteroskedasticity:

$$(i)\ f_{i|q^M c}(i|q^M, c; \theta_0) = N(\alpha_0 + \alpha_1 q^M + \alpha_2 (q^M)^2 + \cdots + \alpha_m (q^M)^m + \alpha_{m+1} c,\ \ e^{\sigma_{01} + \sigma_{02} q^M + \sigma_{03} c}),$$

$$(ii)\ f_{q^A|q^M c}(q^A|q^M, c; \theta_1) = N(\beta_0 + \beta_1 q^M + \beta_2 c,\ \sigma_1^2),$$

$$(iii)\ f_{q^M|q^{IV} c}(q^M|q^{IV}, c; \theta_2) = N(\gamma_0 + \gamma_1 q^{IV} + \gamma_2 c,\ \sigma_2^2),$$

where $\theta_0 = (\alpha_0, \alpha_1, \ldots, \alpha_{m+1}, \sigma_{01}, \sigma_{02}, \sigma_{03})^{\mathrm{T}}$, $\theta_1 = (\beta_0, \beta_1, \beta_2, \sigma_1)^{\mathrm{T}}$, and $\theta_2 = (\gamma_0, \gamma_1, \gamma_2, \sigma_2)^{\mathrm{T}}$. We then test whether augmented regression models with higher-order terms perform better than the classic regression model by investigating log-likelihoods.

We impose Assumption 4 on the conditional density $f_{q^A|q^M c}(q^A|q^M, c; \theta_1)$ in the form of classical measurement error. Abel and Panageas (2020) show that nonclassical measurement error in average q is theoretically possible when there is a stark financial constraint that precludes raising external funds. Considering the fact that analysts follow companies that investors are more interested in, and analysts tend to cover larger firms within an industry (Bhushan, 1989), our IBES-covered firms are unlikely to face such a stark financial constraint. Classical measurement error has zero mean conditional on marginal q and cash flow: $M[f_{q^A|q^M c}(\cdot|q^M, c)] = E[q^A|q^M, c] = E[q^M + \varepsilon|q^M, c] = q^M + E[\varepsilon|q^M, c] = q^M$ where $M[f] = \int_{\mathcal{X}} x f(x) dx$. This leads to the following restrictions on the coefficients of the conditional density $f_{q^A|q^M c}(q^A|q^M, c_j; \theta_1)$: $\beta_0 = 0$, $\beta_1 = 1$, and $\beta_2 = 0$.

In the data, q^{phy} ranges from -0.49 to 37.69, and q^{tot} ranges from -0.18 to 10.70. $\hat{q}^{phy}$ ranges from 0.12 to 38.72, and $\hat{q}^{tot}$ ranges from 0.06 to 12.70. When the dependent variable is i^{phy}, q^M is assumed to range from 0 to 16.87, which is the 95th percentile of q^{phy}. When i^{tot} is the dependent variable, we assume that q^M ranges from 0 to 4.69, which is the 95th percentile of q^{tot}. We then conduct the numerical integral in Equation (9) through the

Gaussian quadrature.

The optimization procedure is repeated with several different plausible initial parameter guesses to minimize the risk of not reaching the global optimum. We use 100 random initial values around the coefficients estimated by OLS. To choose the degree of a polynomial m, we continue to include higher-order q terms until a higher-order term does not increase the log-likelihood significantly.

4. Estimation results

In this section, we test whether a nonlinearity exists in the cross-sectional investment-q relation. We first estimate a linear relationship and then fit a nonlinear relationship to the data using higher-order q terms while correcting for measurement errors using the Hu and Schennach (2008) IV estimator.

4.1. Measurement-error-corrected MLE results

When estimating a regression model with mismeasured regressors, OLS gives biased results. Specifically, in the case of investment regression models, coefficients estimated from OLS are biased because of measurement error in average q, which is used to proxy for marginal q. There are important studies that address this issue. For example, Erickson and Whited (2000, 2002) develop the higher-order moment estimator, and Erickson, Jiang, and Whited (2014) develop the higher-order cumulant estimator to correct for measurement error in average q. These methods deal with measurement error in linear errors-in-variables models like the classic investment equation.

In the case of linear errors-in-variables models, measurement error is separable from a mismeasured regressor. We estimate polynomials to capture nonlinearities, and measurement errors are not separable from mismeasured higher-order qs. Thus, the method that addresses measurement error bias needs to change accordingly. The issue of measurement errors in regressors is not the case of endogeneity anymore in our estimation, so the existence of conventional instrument variables, which satisfy a relevance condition and an exclusion restriction, is not sufficient to control for measurement errors in nonlinear errors-in-variables models.

As described in Section 3, we use the measurement-error-robust maximum likelihood estimation (MLE). The MLE approach can estimate the true parameters of investment equations from the distribution of the observed variables by correcting for measurement errors residing in polynomials in a nonseparable fashion. We stop adding higher-order terms in q when the log-likelihood does not improve significantly. We test whether higher-order terms bring a significant improvement in log-likelihood to the classic regression model via the likelihood-ratio test.

Table 2 contains the results. Columns (1) and (2) show results from regressions with physical investment, lagged physical q terms, and contemporaneous standard cash flow. Columns (3) and (4) report results from regressions using the total measures of investment, q, and cash flow. We consistently find a polynomial of degree 3 when fitting a nonlinear relationship between investment and q. The difference in log-likelihood for each pair of the classic and augmented models is statistically significant at the 1% level. The test statistics of the likelihood-ratio test for the null hypothesis of the classic model specification are extremely high and thus exceed the critical value of the chi-squared distribution with two degrees of freedom. The number of degrees of freedom is determined by the difference in the number of parameters of the nested models. These results show that the added higher-order qs contribute in detecting different investment sensitivities across firms, which cannot be captured by a single q. Stock and Watson (2015) point out that economic data is often smooth, so it is appropriate to choose small orders like 2,3, or 4. Our findings are also consistent with their argument. To summarize, the estimated nonlinear relationships describe the data better than the existing linearity.

[Table 2 here]

Notably, we can observe that total q is better than physical q in explaining the relationship, as it delivers greater log-likelihood values. This corroborates the previous findings in Peters and Taylor (2017) that total q is a superior proxy for investment opportunities.

We also report OLS counterparts in Columns (1) and (3) of Table 3. The results are from OLS regressions of investment on lagged Tobin's q terms, contemporaneous cash flow, and year fixed effects. The MLE results in Table 2 are restated in Columns (2) and (4) to compare the results.

[Table 3 here]

Like from the MLE results, we also find that the augmented models estimated by OLS fit the data better, as evidenced by higher adjusted R^2 values. Also in the OLS results, the total measures in Panel B overwhelm the physical measures in Panel A, as they deliver higher adjusted R^2 values. Compared to the OLS results, the estimated linear relationships from the IV estimator reveal far more sensitive investment-q relations for both measures. On the other hands, the influence of cash flow becomes smaller after correcting for measurement error. The slope coefficients on cash flow are the smallest in the error-corrected polynomials in Column (4). These results reveal the effect of correcting attenuation bias in the OLS estimates of the coefficients on the q proxies through the measurement-error-robust MLE using the IV estimator. For easier interpretation of the results from the different estimators, we visualize the results in Figure 1.

[Figure 1 here]

Panels A and C plot the classic investment model estimated by OLS and MLE in Columns (1) and (2) of Table 3. The MLE estimate for the slope coefficient on physical q in Panel A shows a closer relationship between physical investment and physical q, compared to the OLS estimate. Over the assumed range of marginal q, which is from 0 to the 95th percentile of the observed physical q, the predicted physical investment shows sufficient variation. The same pattern is observed when employing the total measures in Panel C.

The goal of our paper is to estimate different elasticities of investment with respect to cross-sectional values of q. Panels B and D plot the augmented models estimated by OLS and MLE in Columns (3) and (4) of Table 3. The polynomials of degree 3 estimated from OLS do not seem to bring notable dynamics to the cross-sectional variation. The plotted nonlinear relationships look similar to the OLS-estimated linear relationships although they are actually concave. On the other hands, we can observe more dynamic relationships from the MLE results. The estimated nonlinear investment-q relations are consistently S-shaped for both version of q. To summarize, after applying the IV estimator to the empirical models, the association between investment and q becomes tighter, and we can also detect how investment sensitivity evolves in the cross section by adopting higher-order qs.

Figure 2 plots the MLE-estimated polynomials and their first derivatives. The estimated nonlinear physical investment-physical q in Panel A predicts similar levels of investment as predicted by the linear relationship for q below about 5. Investment is then predicted to be higher than predicted by a single physical q for intermediate values of physical q up to about 14. The S-shape finally suggests that firms with higher q do not invest more or even less in the high q region. We find a similar pattern from total investment and total q in Panel C. The augmented regression model predicts similar levels of investment for total q below about 2. Investment is then predicted to be higher than in the linear relationship for intermediate values of total q up to about 4. We can also observe the decreasing level of investment in the high q region, but it is not as conspicuous as in Panel A.

The sensitivity of investment first increases and then decreases in q in Panels B and D. The sensitivity stays around zero for low physical q below 2 and becomes zero again for physical q around 12. It then becomes negative, meaning that the elasticity of physical investment is non-positive for high physical q above 12. The sensitivity of total investment also stays at zero for total q around 4 and then becomes negative. Lastly, when q is low, the response of investment to an improvement in q is not instant as predicted by the classic regression model.

To summarize, as q improves, the sensitivity gradually increases up to some intermediate value of q in the support. After that, it starts to decrease. The elasticity is even non-positive for very high values of q. This indicates that firms with very high q do not invest more than firms with less high q in that region.

4.2. Other proxies of q

Although total q is a superior proxy of the true q, it is not the only measure that proxies for both physical and intangible investment opportunities. We thus test nonlinearities with other versions of total q and compare the results. Woeppel (2021) focuses on the market value of patents to measure internally-generated intangible capital that does not appear on the balance sheet and constructs patent q that incorporates the replacement cost of patent capital. Ewens, Peters, and Wang (2020) re-estimate parameters used in Peters and Taylor (2017) to construct intangible capital using market prices of intangibles of firms that exit publicly traded markets due to acquisitions or bankruptcy. We construct another q proxy using Ewens, Peters, and Wang's (2020) estimate of intangible capital, denoted by EPW q. Using patent q and EPW q, we re-estimate nonlinearities in the data, and the results are reported in Table 4.

[Table 4 here]

The true q is again assumed to range from 0 to the 95th percentile of each proxy. Patent q and its corresponding total investment and total cash flow are used in Columns (1) and (2). EPW q and its version of total investment and total cash flow are employed in Columns (3) and (4). We find polynomials of degree 3 again from both measures. The selected augmented models outperform the classic empirical model for both proxies, as evidenced by higher log-likelihoods. The slope coefficients on cash flow are consistently smaller in the augmented models. Remarkably, EPW q outperforms patent q in explaining the cross-sectional variation in investment as shown in the higher log-likelihoods. The estimated equations are plotted in

Figure 3.

[Figure 3 here]

We can observe S-shapes from both q proxies. Panels A and B show that when patent q is below 0.5, the sensitivity is non-positive. It then starts to gradually increase until $q^{pat} = 4$. After that, the elasticity decreases and stays at zero for patent q above about 8. A similar pattern is observed from EPW q. The investment sensitivity is lower than predicted by the linear relationship for EPW q below 1. It increases until $q^{epw} = 2$ but then decreases. The estimated sensitivity is close to zero for EPW q around 4 and is significantly negative for very high EPW q above about 4.5.

Overall, the estimation results from the IV estimator consistently suggest that the elasticity of investment is very low when q is small, meaning that investment does not increase instantly unlike in a linear relationship when q starts to increase from 0. The sensitivity of investment increases gradually as q improves and decreases in q above an intermediate value in the corresponding support of q, resulting in firms with higher q do not invest more than firms with less high q in a large q region, and this phenomenon completes the observed S-shapes.

We can also figure out which q proxy is the best in depicting the investment-q relation in the cross section based on log-likelihood values. Although all the three new measures of Tobin's q proxy for both physical and intangible investment opportunities and outperform physical q, total q is shown to be the best for our cross-sectional analysis. We thus employ total q and its corresponding measures of investment and cash flow for further analyses.

4.3. Investment-q relation in different periods

We find that the classic regression model augmented with squared and cubed versions of q performs significantly better and consistently derives an S-shape in our full sample. Nevertheless, since the IV estimator cannot accommodate year fixed effects internally, we re-estimate polynomials of degrees 1 and 3 in different periods to see whether we can consistently observe a higher log-likelihood and an S-shape from the augmented model in each subperiod. We fit the classic and augmented models to four different periods, which are 1983–1991, 1992–1999, 2000–2008, and 2009–2017. The assumed range of marginal q is from 0 to the 95th percentile of total q in each subperiod, and the results are reported in Table 5. The augmented models give significantly higher log-likelihoods in all the subperiods, indicating that nonlinearities consistently exist over our sample period.

The visualized results are in Figure 4. We can observe S-shapes from all the subperiods. A decreasing level of investment in a high q region is more evident in the first two subperiods, especially from 1992 to 1999. On the other hand, similar levels of investment are predicted by both the classic and augmented models over the entire assumed ranges of q in the last two subperiods. Nevertheless, the augmented model still provides remarkably different predictions when it comes to investment sensitivity.

Figure 5 plots the first derivatives of the classic and augmented models estimated by the IV estimator. A non-positive investment sensitivity in a high q region is found in all the subperiods. During the earliest period from 1983 to 1991, the response of investment to q is non-positive for low q below 0.5. It then increases but soon decreases and becomes non-positive around $q^{tot} = 2.5$. The sensitivity is negative for q around 3, meaning that firms with higher q invest less in that high q region. We find a similar pattern in the next period. In the original function (Panel B of Figure 4), we cannot find enough variation in investment in the small q zone below 1 due to the low elasticity depicted in Panel B of Figure 5. The sensitivity is non-positive for q above 4. A negative response of investment to q is more evident in this period, which spans q values from 4 to 5.44.

In the recent periods, we can still find such a first convex and then concave relationship between investment and q. However, a significantly negative elasticity of investment is not observed in these periods. The elasticity stays at zero for q above and around 5 in Panel C, meaning that firms with higher q do not invest more in that high q zone. Moreover, investment is not sensitive to low q in the latest period. We find that investment is only sensitive to a limited range of q in Panel D. The elasticity stays at zero for q below 1 and above and around 3. This result indicates that firms do not increase their investments immediately when q is low and do not further increase investments when q reaches a high value like 3. We also find consistent results from untabulated results using the other measures of q.

The evidence so far supports nonlinearities between investment and q in the cross section, which indicates that the true investment-q relation can be illustrated more precisely by higher-order terms in q with the existing regressors. Specifically, an S-shaped relationship is found regardless of the q proxy, and the shape is robust in the entire sample period. The S-shape suggests that unlike in a linear relationship, firms with low q are not predicted to increase their investments instantly, and firms with high q are not predicted to further increase investments as q grows over an intermediate value. Moreover, firms with higher q are predicted to invest even less in some cases. Most importantly, this investment tendency in the cross section cannot be found in OLS results. We further discuss our measurement-error-corrected MLE results in Section 5.

4.4. Cumulant estimator results

The classic investment model can also be correctly measured by the cumulant estimator of Erickson, Jiang, and Whited (2014). The cumulant estimator estimates classical linear errors-in-variables models using information in the higher-order cumulants of observable variables to identify the coefficients. We compare our estimation of the classic model using the IV estimator with the results from the cumulant estimator to see whether the IV estimator corrects measurement error bias in a way consistent with the cumulant estimator. The OLS and cumulant results using the full sample of the total measures of investment, q, and cash flow are reported in Columns (1) and (2) of Table 6, respectively.[5] The original data are

[5]Following Peters and Taylor (2017) and Woeppel (2021), we use the third-order cumulant estimator. All Sargan-Hansen J statistics associated with the third- and higher-order cumulants (untabulated) reject the

within-year transformed to control for year-fixed effects.

Similar to Column (2) of Panel B of Table 3, the cumulant estimator produces a higher coefficient on q and a lower coefficient on cash flow after correcting attenuation bias. However, the estimates are slightly different for the q slope and largely different for the cash flow slope. Possible reasons for the discrepancy are different estimators and the consideration of year fixed effects. Nevertheless, conditioning on cash flow, our estimation of the q slope gives a similar investment-q sensitivity over the assumed range of the true q as in the cumulant-estimated classic regression model, indicating the validity of the IV estimator in addressing measurement error in average q. The same pattern of correcting attenuation bias can be observed from both estimators in the subsample analysis as well. Compared to the OLS estimates (Columns (1) and (3) of Table 7), both the IV estimator (Columns (1) and (3) of Table 5) and the cumulant estimator (Columns (2) and (4) of Table 7) produce larger coefficients on q and smaller coefficients on cash flow. Especially after 2000, the estimated investment-q sensitivities are very close to each other, conditioning on cash flow, which corroborates the validity of the IV estimator in estimating the classic investment regression. More importantly, the necessity of the IV estimator is related to our goal to estimate a nonlinear errors-in-variables model with the aid of an instrument. Thus, the availability of a valid instrument is the key in our estimation.

As we discuss in Section 2, analyst q is a valid instrument for the identification of the bias-corrected investment-q relation in the cross section thanks to the independence of data source owned by analysts. Analyst q is thus expected to be uncorrelated with measurement error in the existing market-based measure of q (Abel and Eberly, 2002). Our next question is whether analyst q can be more than just an instrument. In other words, we investigate whether analyst q itself can be used as a sole q proxy. Also, the instrument is the main regressor of the third conditional density in Equation (8).

We re-estimate the classic investment regression using the analyst-based measure of total

null hypothesis that the overidentifying restrictions are valid.

q and the cumulant estimator to see how well analyst total q proxies for the true q. In Table 6, we can observe that analyst total q leads to a higher adjusted R^2 and a higher ρ^2, which is the coefficient of determination in the hypothetical regression of investment on the true q and cash flow. Moreover, analyst total q also produces a higher τ^2, which is the coefficient of determination in the hypothetical regression of the true q on a q proxy, indicating that analyst total q is not only a valid instrument but also a better proxy of q than the market-based measure of total q. Consistent results are found when using the other analyst-based measures of q and also when using the fourth- and higher-order cumulant estimators (untabulated).

5. Discussion

We find evidence that there exist heterogeneous investment sensitivities across firms. According to the estimated nonlinear investment-q relation, firms with low q have similarly low investment elasticities. There is little variation in investment over the low q zone. As q improves, the sensitivity gradually increases. It then starts to decrease after an intermediate value of q, although investment keeps increasing until q reaches a high value in an assumed support. After that, the elasticity becomes non-positive, and in some cases, investment is predicted to decrease as q keeps growing over a high value. In other words, firms with higher q do not invest more over the high q zone. In this section, we discuss our results from both theoretical and empirical perspectives.

5.1. Capital heterogeneity

We can find a theoretical basis for the S-shaped investment-q relation from Abel and Eberly (1994), Abel and Eberly (2002), Eberly (1997), and Barnett and Sakellaris (1998). The idea of a nonlinear relationship between investment and q originates from Abel and Eberly (1994). Abel and Eberly (1994) extend the standard q theory of investment of

Hayashi (1982) by incorporating fixed costs and irreversibility of investment (i.e., different purchase and resale prices of capital goods) in the adjustment cost function.

Abel and Eberly (1994) show that if the augmented adjustment cost function is nondifferentiable at the level of investment equal to zero due to the presence of fixed costs and irreversibility of investment, there can be a region in which the optimal investment stays at zero but does not satisfy the first-order condition for a range of q. As a result, investment behavior alternates between responsive and irresponsive regimes in response to q, which implies a nonlinearity in the investment-q relation. Investment is still a nondecreasing function of q, but its sensitivity to q can vary across the regions above and below the regime of irresponsive investment behavior in this extended framework.

Barnett and Sakellaris (1998) empirically test this idea by allowing the relationship to vary across regimes defined by the level of average q. However, they emphasize that a direct test of Abel and Eberly's (1994) model is difficult in practice because there are few observations of zero and negative investments when using the Compustat item *capx* as the gross investment variable. They instead show that the model can further be extended by embracing another regime. Suppose that an additional proportional cost incurs when the firm invests beyond a threshold rate of investment, say $I/K = \theta$. If θ equals the depreciation rate of capital stock, say δ, investment beyond δK is for expansion, and the price of capital for expansion can be higher than that for replacement. If θ is the usual rate of investment, and if the firm invests beyond that, an additional proportional cost can incur in the form of overtime wage for installing additional capital goods. As a result, the augmented adjustment cost function has a kink at $I/K = \theta$, and there comes another region of insensitivity, which results in three regimes for positive investments.

Barnett and Sakellaris then estimate nonlinear specifications that include up to a cubic term in average q or in log of average q in each regime to take account of the effect of higher-order terms in q.[6] The identified investment-q relation is convex for low values of

[6] The three regimes are defined by threshold parameters, say ω_h and ω_l. These are nuisance parameters that are not identified under the null hypothesis of only one regime. These parameters are identified by

q and concave for intermediate and high values of q, i.e., an S-shape, with no regime of insensitivity, and the relationship is shown to be better explained under the three regimes.

Abel and Eberly (2002) and Eberly (1997) specify the parametric form of the augmented adjustment cost function and show that a firm starts to invest when its q exceeds an upper threshold, say $\bar{q}$. In other words, a firm decides to invest when the total return of investment is high enough to at least cover the total costs. Abel and Eberly find significant nonlinearities by allowing for either a nonquadratic adjustment cost function or capital heterogeneity.[7] A concave relationship between investment and q is estimated in their homogeneous capital model with nonquadratic adjustment costs. On the other hand, their heterogeneous capital model, which enables the firm to invest in heterogeneous capital goods, provides rationales for the S-shape found in our paper. In the heterogeneous capital model, firms increase their investments both in the scale (intensive margin) and in the number of types (extensive margin) as q improves. However, when q is low, firms choose to invest little and in few types of capital. Thus, the model predicts a stronger response of investment after q improves beyond low values, and such a strong sensitivity of investment can be interpreted as an outcome of both intensive and extensive margins being reflected in investment decisions. However, the extensive margin is gradually depleted as q keeps growing. This phenomenon is manifested in the reduced response of investment in the intermediate and high q regions in our results.

Eberly (1997) estimates the homogeneous capital model and finds a convex investment-q relation in 6 out of 11 countries including the US. In the heterogeneous capital model, Eberly shows that a degenerate extensive margin can be rejected, so linearity can be rejected in 9 out of 11 countries and that a degenerate intensive margin can also be rejected in 4 out of the 9 countries including the US. This means that both margins play statistically significant roles

estimating an alternative threshold model using the technique developed by Hansen (1996). See Barnett and Sakellaris (1998) for details.

[7]Even if the adjustment cost function is quadratic, so the investment-q relation is linear for any given type of capital, the aggregation across different types of capital with different investment thresholds can make the overall relationship nonlinear.

in deciding investment spending in those four countries, which predicts a convex relationship between investment and q. The contradictory results between Abel and Eberly (2002) and Eberly (1997) can be due to different samples. Abel and Eberly use an unbalanced panel of US firms, while Eberly uses a balanced panel of international firms. Eberly emphasizes that sample selection is a critical issue in identifying nonlinearities because the distribution of q determines where to be examined and one's conclusion can be different accordingly. On the other hand, it is less important in a linear relationship because the relationship is assumed to be the same across different values of q.

The S-shape in the cross section can be interpreted from the perspective of the heterogeneous capital model. For firms with low q (e.g., below 1 in Panels C and D of Figure 2), their investment levels are clustered around 0.1 with little variation. This is consistent with the prediction in the heterogeneous capital model that firms choose to invest little and in few types of capital when q is low. After the low q zone, the elasticity of investment first increases and then decreases as q keeps improving, resulting in the decreasing response of investment to q over an intermediate value (e.g., 2 in Panels C and D of Figure 2). This prediction is also consistent with the model in which firms invest both in the scale and in the number of types of capital goods as q grows and then begin to invest mostly in the scale because only a few types of capital remain when q is high.

5.2. Industrial organization (IO) q

Interestingly, in our results, the elasticity of investment not only decreases but also becomes non-positive and even negative for very large q (e.g., $q^{tot} > 4$ in Panel D of Figure 2). This phenomenon results in investments clustered around 0.4 with little variation in the high q zone (e.g., Panel C of Figure 2). This indicates that the intensive margin can also be depleted, and after that, firms reduce their investments although q increases. The response of investment to q for very high q can be significantly negative, as evidenced by the confidence bands in Figure 2, for example. Although depletion of the intensive margin is not considered in the heterogeneous capital model, it is plausible because firms cannot invest infinitely in the remained few capital goods. There are also theoretical and empirical perspectives that view q as a proxy for rents than investment opportunities for firms with large q.

In the industrial organization literature, q is mostly used as a measure of monopoly and Ricardian rents (e.g., Stigler, 1964; Lindenberg and Ross, 1981; Montgomery and Wernerfelt, 1988). Lee, Shin, and Stulz (2021) predict that if a firm has high q because of rents, the firm would not increase its size through investment because it wants its unique assets to be in short supply. If this is the case, Tobin's q is called as IO q. Lee, Shin, and Stulz (2021) find that both the industry-level and firm-level sensitivities of investment to q collapse in a recent period (1997–2014). In the meantime, capital flows out of high q industries because high q industries return equity capital to investors. They argue that in the case of large old firms, q capitalizes rents from their market power, and those with higher q have more cash flow to payout. As a result, paying out to investors is optimal if there are only poor projects left, so equity capital flows out of high q industries.

In our results, the decreasing and non-positive sensitivity of investment is not restricted to recent periods. We can observe it from the entire period. The negative investment sensitivity is consistent with the idea of IO q. After the extensive margin is depleted, firms start to focus on the intensive margin. However, the intensive margin is eventually depleted too as firms keep investing in a few types of capital remained. High q firms with no margin to invest now choose to pay out rents from their past investment to investors instead of investing in poor projects.

We run OLS regressions of net payout on lagged total q, contemporaneous total cash flow, and year fixed effects. We compute net payout as cash dividends (Compustat item *dv*) plus the purchase of common and preferred stock (Compustat item *prstkc*) minus the sale of common and preferred stock (Compustat item *sstk*), scaled by lagged total capital. The results are reported in Table 8. As shown in Lee, Shin, and Stulz (2021), firms with higher q consistently pay out and repurchase more during the entire period in our data, indicating that a firm with the largest q pays out the most. Considering that cash for dividends and repurchases mainly comes from internal cash and leads to reductions in capital expenditures and R&D expenses (e.g., Wang, Yin, and Yu, 2021), the negative response of investment to very large q can be explained by IO q.

6. Concluding remarks

The neoclassical investment-q theory has been widely tested on its performance in explaining corporate investment behavior, and most studies estimate linear relationships. Our paper starts with a simple question: what if the true optimal investment behavior is more complicated than that described by a linear relationship? According to Hayashi (1982), the optimal investment is an unknown function of marginal q, which indicates that the true investment-q is not necessarily linear. Moreover, when it comes to the cross-sectional relationship, firms with different q characteristics are likely to have non-homogeneous sensitivities of investment due to heterogeneous sets of capital to invest (e.g., Abel and Eberly, 2002). We augment the classic investment regression with higher-order terms in q and fit the augmented model to the cross section to capture potential nonlinearities in the investment-q relation while correcting for linear and nonlinear measurement errors in q terms. We find that investment is an S-shaped function of q. Firms with higher q invest more in most part of the support of q, but the elasticity of investment is very low for firms with low q. The investment sensitivity decreases for intermediate-high values of q and is predicted to be negative for firms with very large q. This implies that it is unlikely that an improvement in q caused by a policy change, for example, a corporate tax cut, leads firms with small or large q to sufficiently increase their investments. We consistently find S-shapes from the entire sample period and from all proxies of q.

Our sample is constructed based on Compustat firms covered by IBES from 1982 to 2017, which is to construct a repeated measurement of average q using analysts' forecasts of earnings and long-term growth. As a result, the firms in our sample might not represent the average US public firm, so one must be careful with extrapolation of our results to the rest of the firms in Compustat that are not included in our sample. Nevertheless, our results still have important implications because our findings are about investment decisions made by firms that represent 80% of the total market capitalization of the US firms that are available for a general corporate investment study during the sample period. This means that our results focus on investment behavior of firms that mainly drive investment spending in the US economy.

References

Abel, A. B., 2018. The effects of q and cash flow on investment in the presence of measurement error. Journal of Financial Economics 128, 363–377.

Abel, A. B., Eberly, J. C., 1994. A unified model of investment under uncertainty. American Economic Review 84, 1369–1384.

Abel, A. B., Eberly, J. C., 2002. Investment and q with fixed costs: An empirical analysis. University of Pennsylvania, mimeo, revised .

Abel, A. B., Eberly, J. C., 2011. How Q and cash flow affect investment without frictions: An analytic explanation. Review of Economic Studies 78, 1179–1200.

Abel, A. B., Panageas, S., 2020. An analytic framework for interpreting investment regressions in the presence of financial constraints. No. w26898. National Bureau of Economic Research .

Almeida, H., Campello, M., 2007. Financial constraints, asset tangibility, and corporate investment. Review of Financial Studies 20, 1429–1460.

Almeida, H., Campello, M., Galvao, A. F., 2010. Measurement errors in investment equations. Review of Financial Studies 23, 3279–3328.

Almeida, H., Campello, M., Weisbach, M. S., 2004. The cash flow sensitivity of cash. Journal of Finance 59, 1777–1804.

Andrei, D., Mann, W., Moyen, N., 2019. Why did the q theory of investment start working? Journal of Financial Economics 133, 251–272.

Ağca, Ş., Mozumdar, A., 2017. Investment–cash flow sensitivity: Fact or fiction? Journal of Financial and Quantitative Analysis 52, 1111–1141.

Barnett, S. A., Sakellaris, P., 1998. Nonlinear response of firm investment to Q: Testing a model of convex and non-convex adjustment costs. Journal of Monetary Economics 42, 261–288.

Bhushan, R., 1989. Firm characteristics and analyst following. Journal of Accounting and Economics 11, 255–274.

Cleary, S., 1999. The relationship between firm investment and financial status. Journal of Finance 54, 673–692.

Cummins, J. G., Hassett, K. A., Oliner, S. D., 2006. Investment behavior, observable expectations, and internal funds. American Economic Review 96, 796–810.

De Bondt, W. F. M., Thaler, R. H., 1990. Do security analysts overreact? American Economic Review 80, 52–57.

Easterwood, J. C., Nutt, S. R., 1999. Inefficiency in analysts' earnings forecasts: Systematic misreaction or systematic optimism? Journal of Finance 54, 1777–1797.

Eberly, J. C., 1997. International evidence on investment and fundamentals. European Economic Review 41, 1055–1078.

Erickson, T., Jiang, C. H., Whited, T. M., 2014. Minimum distance estimation of the errors-in-variables model using linear cumulant equations. Journal of Econometrics 183, 211–221.

Erickson, T., Whited, T. M., 2000. Measurement error and the relationship between investment and q. Journal of Political Economy 108, 1027–1057.

Erickson, T., Whited, T. M., 2002. Two-step gmm estimation of the errors-in-variables model using high-order moments. Econometric Theory 18, 776–799.

Erickson, T., Whited, T. M., 2012. Treating measurement error in Tobin's q. Review of Financial Studies 25, 1286–1329.

Ewens, M., Peters, R., Wang, S., 2020. Measuring intangible capital with market prices. Working Paper .

Fazzari, S., Hubbard, R. G., Petersen, B. C., 1988. Financing constraints and corporate investment. Brookings Papers on Economic Activity 1, 141–195.

Firpo, S., Galvao, A. F., Song, S., 2017. Measurement errors in quantile regression models. Journal of Econometrics 198, 146–164.

Gilchrist, S., Himmelberg, C. P., 1995. Evidence on the role of cash flow for investment. Journal of Monetary Economics 36, 541–572.

Gourio, F., Rudanko, L., 2014. Customer capital. Review of Economic Studies 81, 1102–1136.

Graham, J. R., 1996. Proxies for the corporate marginal tax rate. Journal of Financial Economics 42, 187–221.

Hansen, B. E., 1996. Inference when a nuisance parameter is not identified under the null hypothesis. Econometrica 64, 413–430.

Hayashi, F., 1982. Tobin's marginal q and average q: A neoclassical interpretation. Econometrica 50, 213–224.

Himmelberg, C. P., Petersen, B. C., 1994. R&D and internal finance: A panel study of small firms in high-tech industries. Review of Economics and Statistics 76, 38–51.

Hoberg, G., Maksimovic, V., 2021. Product life cycles in corporate finance. Available at SSRN 3182158 .

Hu, Y., Schennach, S. M., 2008. Instrumental variable treatment of nonclassical measurement error models. Econometrica 76, 195–216.

Jorgenson, D. W., 1963. Capital theory and investment behavior. American Economic Review 53, 247–259.

Kaplan, S. N., Zingales, L., 1997. Do investment-cash flow sensitivities provide useful measures of financing constraints? Quarterly Journal of Economics 112, 169–215.

Lee, D. W., Shin, H., Stulz, R. M., 2021. Why does equity capital flow out of high Tobin's q industries? Review of Financial Studies 34, 1867–1906.

Lewellen, J., Lewellen, K., 2016. Investment and cash flow: New evidence. Journal of Financial and Quantitative Analysis 51, 1135–1164.

Lindenberg, E. B., Ross, S. A., 1981. Tobin's q ratio and industrial organization. Journal of Business 54, 1–32.

Montgomery, C. A., Wernerfelt, B., 1988. Diversification, ricardian rents, and Tobin's q. The RAND Journal of Economics 19, 623–632.

Peters, R. H., Taylor, L. A., 2017. Intangible capital and the investment-q relation. Journal of Financial Economics 123, 251–272.

Song, S., 2015. Semiparametric estimation of models with conditional moment restrictions in the presence of nonclassical measurement errors. Journal of Econometrics 185, 95–109.

Stigler, G. J., 1964. A theory of oligopoly. Journal of Political Economy 72, 44–61.

Stock, J. H., Watson, M. W., 2015. Introduction to Econometrics, Update, Global Edition, 3rd Edition. Pearson Education.

Tobin, J., 1969. A general equilibrium approach to monetary theory. Journal of Money, Credit and Banking 1, 15–29.

Wang, Z., Yin, Q. E., Yu, L., 2021. Real effects of share repurchases legalization on corporate behaviors. Journal of Financial Economics 140, 197–219.

Whited, T. M., 1992. Debt, liquidity constraints, and corporate investment: Evidence from panel data. Journal of Finance 47, 1425–1460.

Woeppel, M., 2021. Using patent capital to estimate Tobin's *q*. Available at SSRN 3420322
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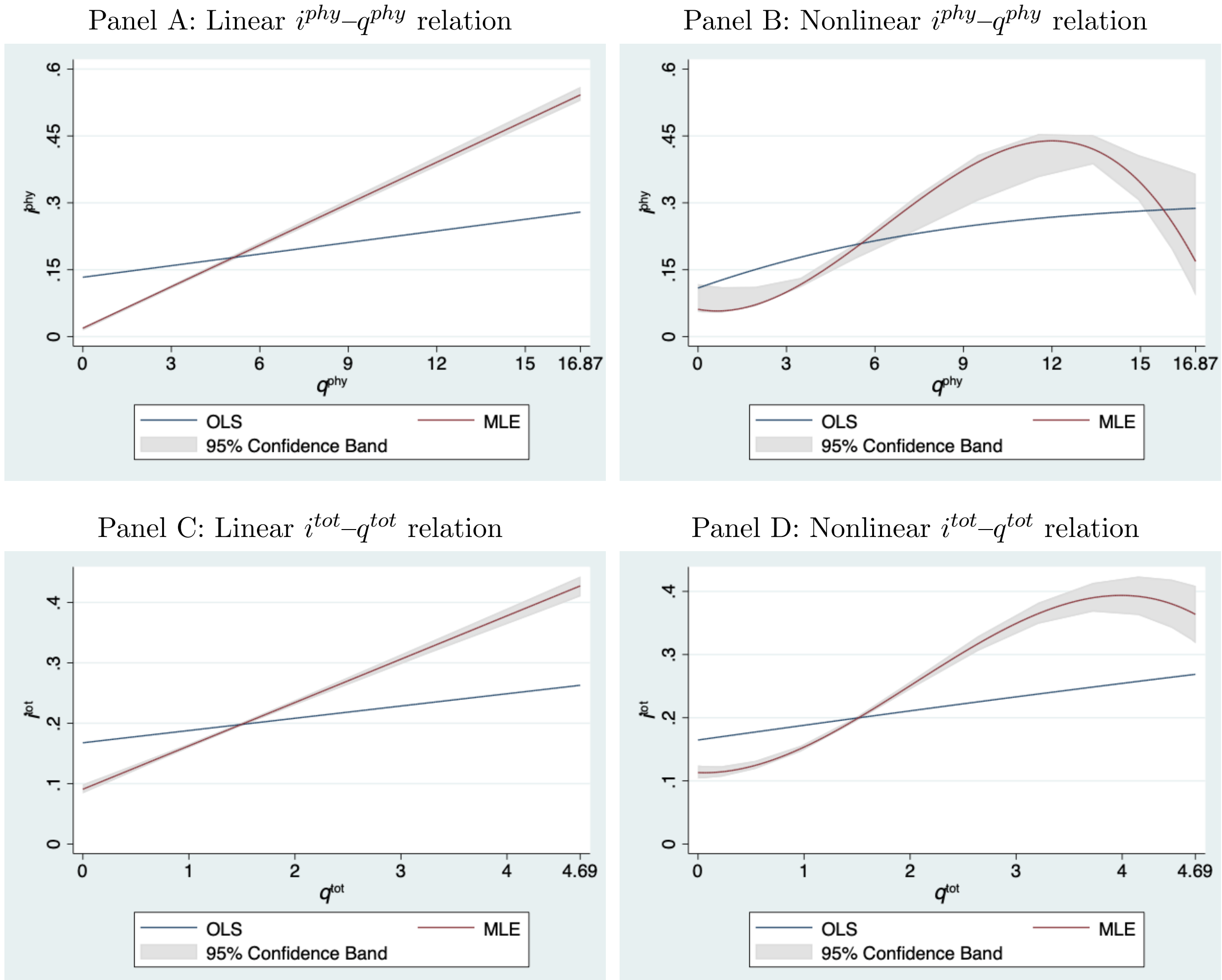


Figure 1. Investment-q relation

This figure presents investment as a function of q. The classic and augmented investment models estimated by OLS and measurement-error-robust MLE using the Hu and Schennach (2008) IV estimator in Table 3 are plotted. Panels A and C (B and D) plot the estimated linear (nonlinear) relationship between investment and q. 95% bootstrapped confidence bands are plotted. Physical (total) investment and physical (total) q are the variables in Panels A and B (C and D).

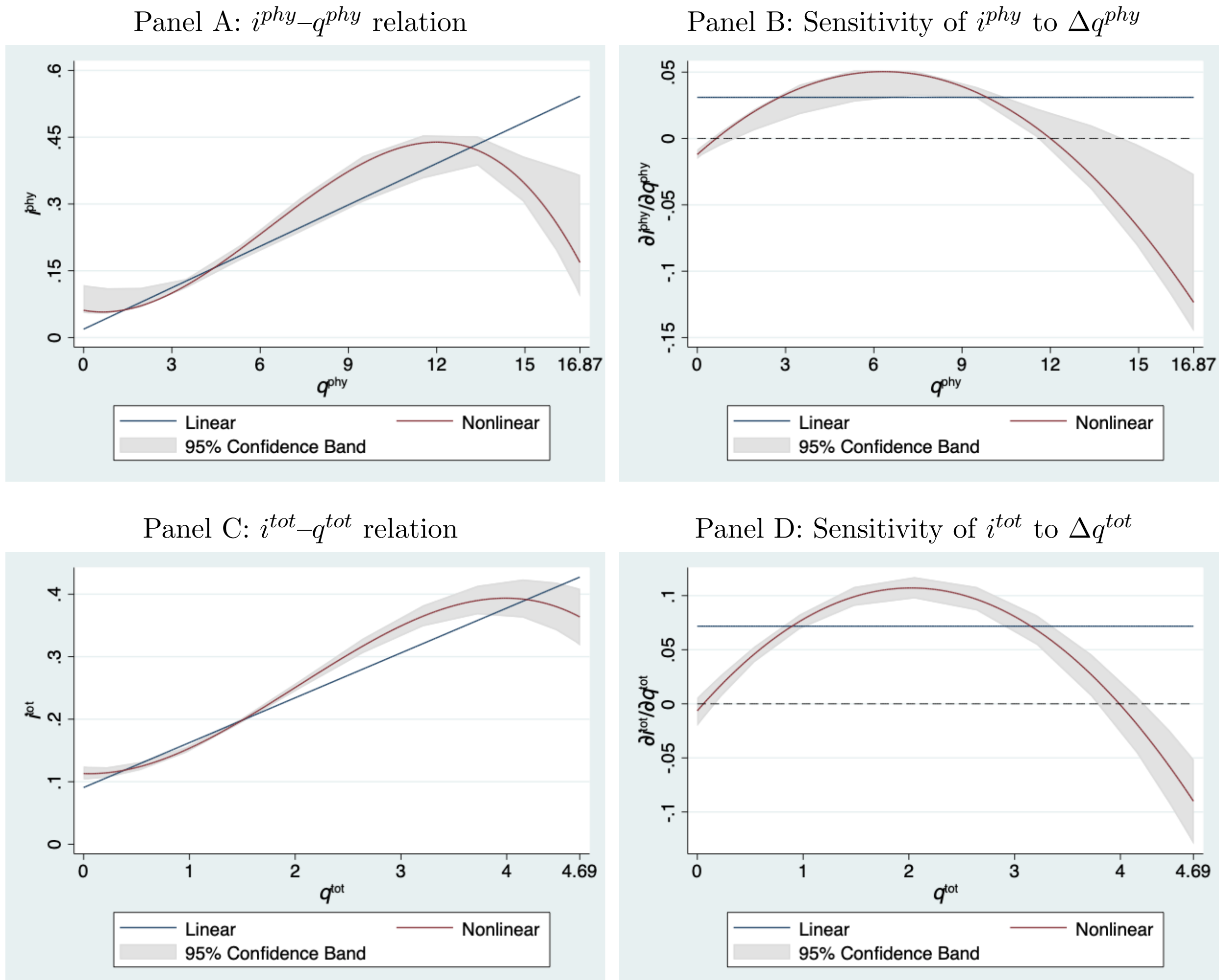


Figure 2. The elasticity of investment with respect to $\boldsymbol{q}$

This figure presents the relationship between investment and q. The classic and augmented investment models estimated by measurement-error-robust MLE using the Hu and Schennach (2008) IV estimator in Table 2 are plotted in Panels A and C. The first derivatives with respect to q from the estimated classic and augmented investment models are plotted in Panels B and D. 95% bootstrapped confidence bands are plotted. Physical (total) investment and physical (total) q are the variables in Panels A and B (C and D).

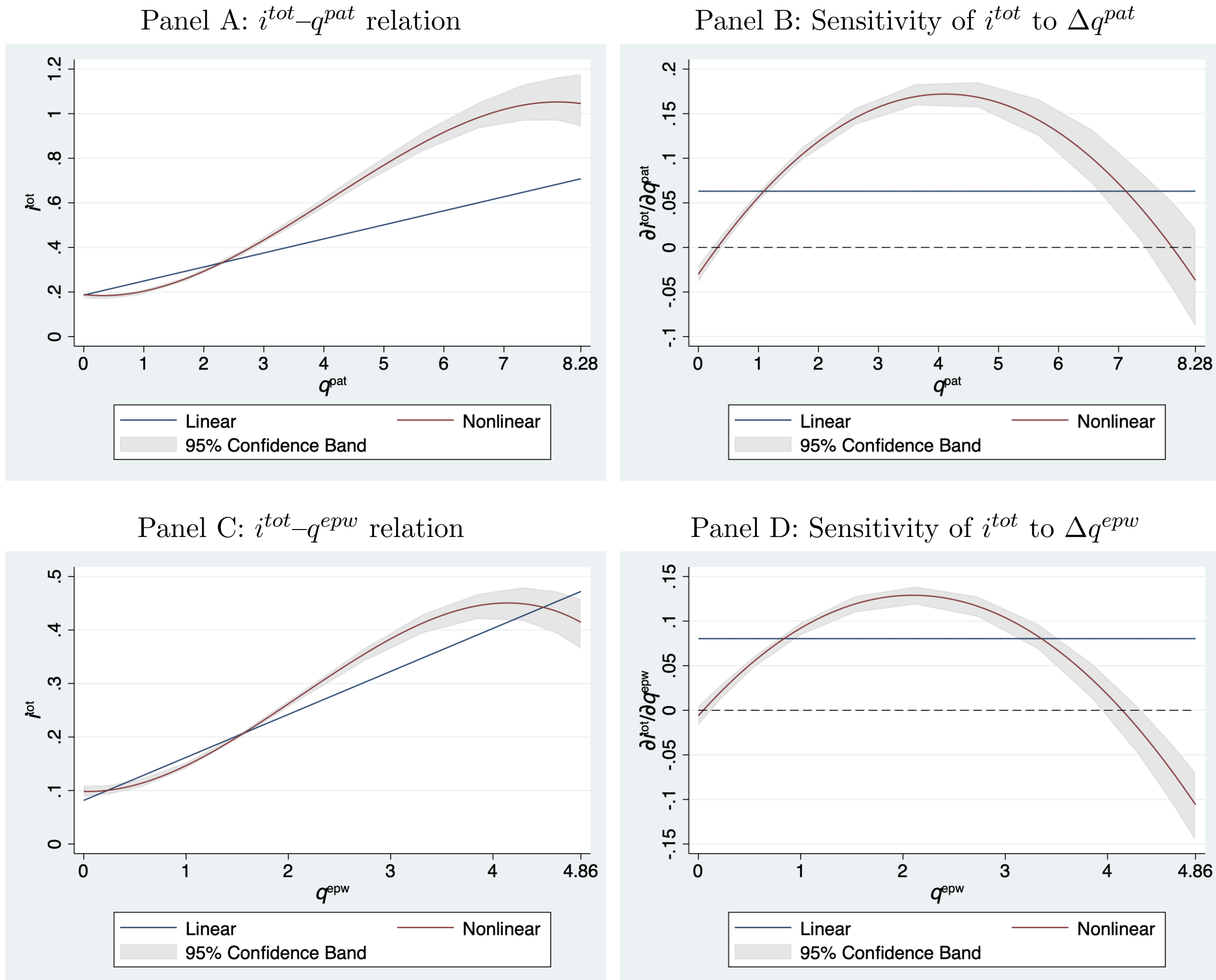


Figure 3. Investment-patent q/EPW q relation

This figure presents the relationship between investment and other proxies of q, which are patent q and EPW q. The classic and augmented investment models estimated by measurement-error-robust MLE using the Hu and Schennach (2008) IV estimator in Table 4 are plotted in Panels A and C. The first derivatives with respect to q from the estimated classic and augmented investment models are plotted in Panels B and D. 95% bootstrapped confidence bands are plotted. Patent q (EPW q) and its corresponding total investment are the variables in Panels A and B (C and D).

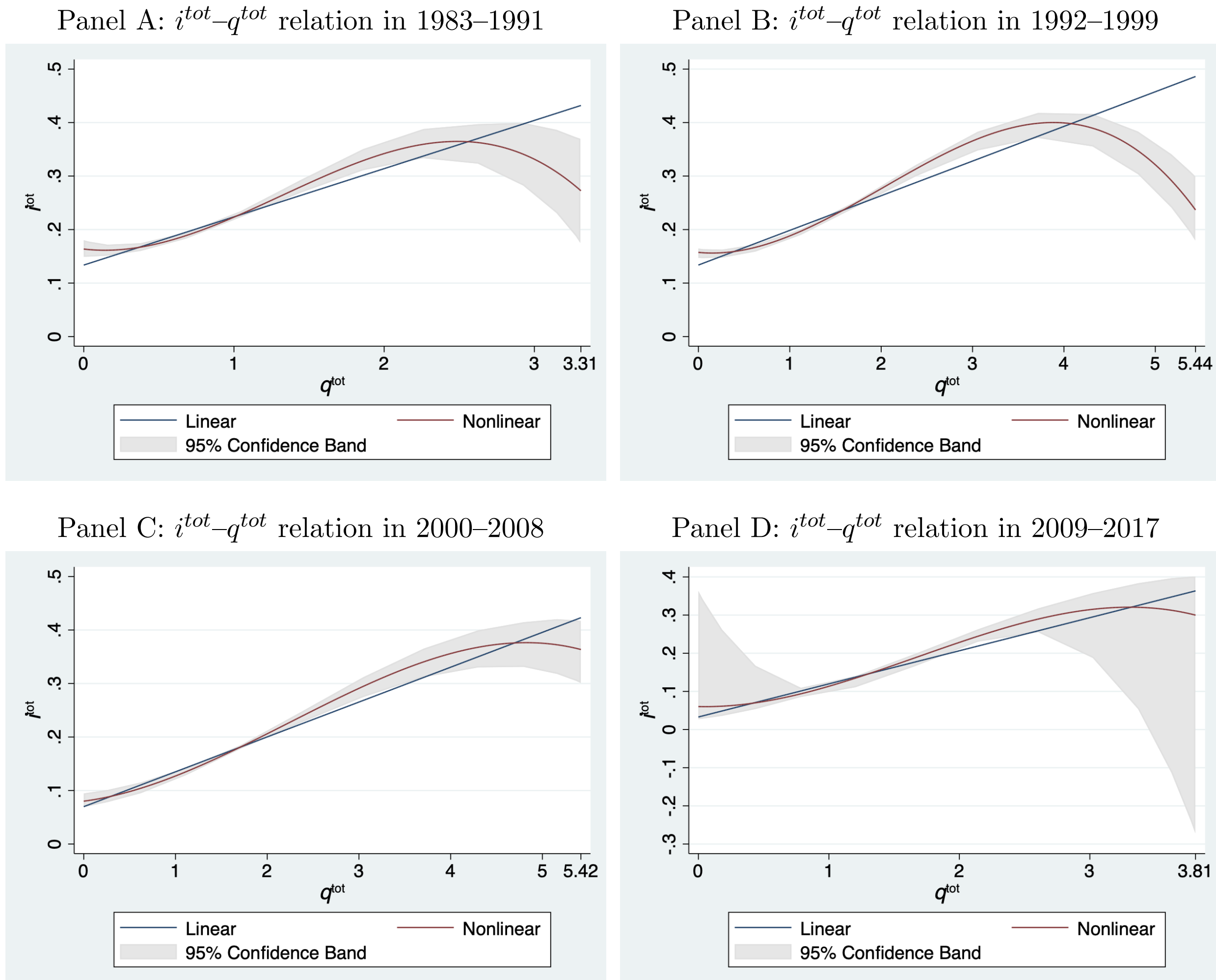
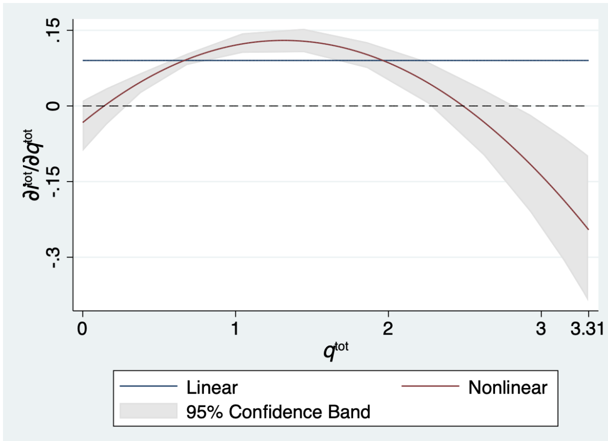


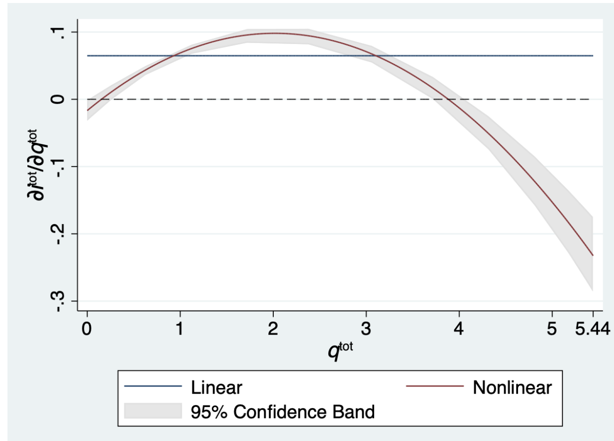
Figure 4. Investment-q relation in different periods

This figure presents investment as a function of q in different periods. The classic and augmented investment models estimated by measurement-error-robust MLE using the Hu and Schennach (2008) IV estimator in Table 5 are plotted. 95% bootstrapped confidence bands are plotted. Total investment and total q are the variables.

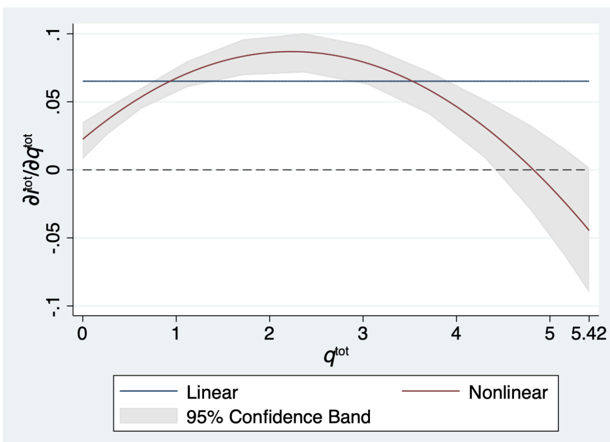
Panel A: Sensitivity of i^{tot} to Δq^{tot} in 1983–1991



Panel B: Sensitivity of i^{tot} to Δq^{tot} in 1992–1999



Panel C: Sensitivity of i^{tot} to Δq^{tot} in 2000–2008



Panel D: Sensitivity of i^{tot} to Δq^{tot} in 2009–2017

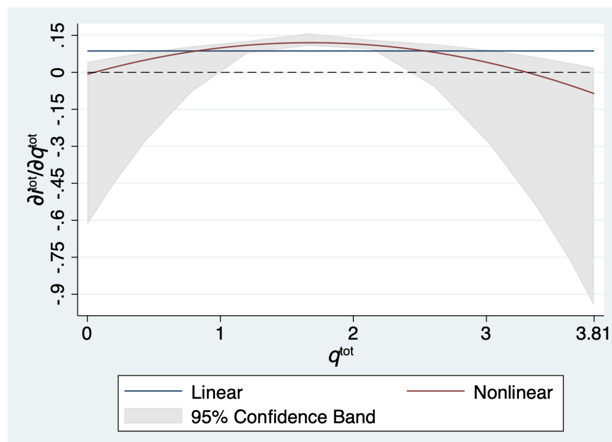


Figure 5. The elasticity of investment with respect to q in different periods

This figure presents the first derivatives with respect to q in different periods from the classic and augmented investment models estimated by measurement-error-robust MLE using the Hu and Schennach (2008) IV estimator. 95% bootstrapped confidence bands are plotted. Total investment and total q are the variables.

Table 1. Summary statistics

This table presents summary statistics on the annual Compustat sample covered by IBES from 1983 to 2017. Physical investment is CAPX divided by lagged physical capital (i.e., gross PP&E). Total investment is CAPX plus intangible investment (i.e., R&D expense plus 30% of SG&A expense), divided by lagged total capital (i.e., the sum of physical capital and Peters and Taylor's (2017) estimate of intangible capital). The numerator of each market-based (i.e., non analyst-based) measure of q is the market value of equity plus the book value of debt minus current assets. The numerator of each analyst-based measure of q is the analyst value of equity plus the book value of debt minus current assets. The denominators of physical q and analyst physical q are physical capital. The denominators of total q and analyst total q are total capital. The denominators of patent q and analyst patent q are the sum of physical capital and Woeppel's (2021) estimate of intangible capital (i.e., patent capital plus on-balance sheet intangible capital). The denominators of EPW q and Analyst EPW q are the sum of physical capital and Ewens, Peters, and Wang's (2020) estimate of intangible capital. Standard cash flow is income before extraordinary items plus depreciation, divided by lagged physical capital. Total cash flow is income before extraordinary items plus depreciation plus tax-adjusted intangible investment, scaled by lagged total capital. Net payout is cash dividends plus the purchase of common and preferred stock minus the sale of common and preferred stock, scaled by lagged total capital. All ratios are winsorized at the 1st and 99th percentiles.

	Mean	Standard deviation	P5	Median	P95	Observations
Physical investment	0.169	0.158	0.035	0.118	0.480	58,796
Total investment	0.196	0.146	0.049	0.157	0.488	58,796
Physical q	4.119	6.359	0.172	1.748	16.872	58,796
Analyst physical q	5.685	7.412	0.409	2.832	22.456	58,796
Total q	1.399	1.720	0.087	0.865	4.693	58,796
Analyst total q	2.020	2.199	0.239	1.312	6.531	58,796
Patent q	2.183	3.375	0.139	1.089	8.282	58,796
Analyst patent q	3.274	4.573	0.312	1.710	12.411	58,796
EPW q	1.459	1.764	0.097	0.908	4.855	58,796
Analyst EPW q	2.116	2.268	0.254	1.389	6.791	58,796
Standard cash flow	0.288	0.415	-0.135	0.205	1.069	58,796
Total cash flow	0.190	0.150	0.014	0.163	0.479	58,796
Net payout	0.029	0.054	0.000	0.006	0.133	58,796

Table 2. Measurement-error-corrected maximum likelihood estimation results

This table reports the classic and augmented investment models estimated by measurement-error-robust MLE using the Hu and Schennach (2008) IV estimator. Physical investment, lagged physical q (q^{phy}), and contemporaneous standard cash flow (c) are used in Columns (1) and (2). Total investment, lagged total q (q^{tot}), and contemporaneous total cash flow (c^{tot}) are used in Columns (3) and (4). Block bootstrapped standard errors treating each firm as one block are computed based on 500 replications and are reported in parentheses.

	(1)	(2)	(3)	(4)
	Physical investment		Total investment	
q^{phy}	0.0310	-0.0121		
	(0.0005)	(0.0142)		
c	-0.0173	0.0013		
	(0.0059)	(0.0043)		
$(q^{phy})^2$		0.0099		
		(0.0035)		
$(q^{phy})^3$		-0.0005		
		(0.0002)		
q^{tot}			0.0718	-0.0065
			(0.0024)	(0.0277)
c^{tot}			0.3574	0.3155
			(0.0169)	(0.0223)
$(q^{tot})^2$				0.0561
				(0.0159)
$(q^{tot})^3$				-0.0092
				(0.0026)
Log-likelihood	-122814.89	-122057.61	-37838.07	-37581.75
LR test statistic		1514.56		512.64
Observations	58,796	58,796	58,796	58,796

Table 3. Comparing OLS and measurement-error-corrected MLE results

This table reports the classic and augmented investment models estimated by OLS and measurement-error-robust MLE using the Hu and Schennach (2008) IV estimator. Physical investment, lagged physical q (q^{phy}), and contemporaneous standard cash flow (c) are used in Panel A. Total investment, lagged total q (q^{tot}), and contemporaneous total cash flow (c^{tot}) are used in Panel B. OLS results are from regressions of investment on lagged Tobin's q, contemporaneous cash flow, and year fixed effects. Measurement-error-corrected MLE results are restated from Table 2. Standard errors clustered by firm from OLS and block bootstrapped standard errors from the IV estimator are reported in parentheses.

	(1)	(2)	(3)	(4)
Panel A: Physical investment	OLS	MLE	OLS	MLE
q^{phy}	0.0087	0.0310	0.0233	-0.0121
	(0.0003)	(0.0005)	(0.0009)	(0.0142)
c	0.0433	-0.0173	0.0341	0.0013
	(0.0034)	(0.0059)	(0.0034)	(0.0043)
$(q^{phy})^2$			-0.0011	0.0099
			(0.0001)	(0.0035)
$(q^{phy})^3$			0.0000	-0.0005
			(0.0000)	(0.0002)
Adjusted R^2	0.2071		0.2207	
Log-likelihood		-122814.89		-122057.61
LR test statistic				1514.56
Panel B: Total investment	OLS	MLE	OLS	MLE
q^{tot}	0.0203	0.0718	0.0235	-0.0065
	(0.0010)	(0.0024)	(0.0026)	(0.0277)
c^{tot}	0.4285	0.3574	0.4251	0.3155
	(0.0092)	(0.0169)	(0.0093)	(0.0223)
$(q^{tot})^2$			-0.0002	0.0561
			(0.0008)	(0.0159)
$(q^{tot})^3$			-0.0000	-0.0092
			(0.0001)	(0.0026)
Adjusted R^2	0.4316		0.4319	
Log-likelihood		-37838.07		-37581.75
LR test statistic				512.64
Observations	58,796	58,796	58,796	58,796

Table 4. Other proxies of q: patent q and EPW q

This table reports the classic and augmented investment models estimated by measurement-error-robust MLE using the Hu and Schennach (2008) IV estimator. Total investment (scaled by the denominator of lagged patent q), lagged patent q (q^{pat}), and total cash flow (c^{tot}) (scaled by the denominator of lagged patent q) are used in Columns (1) and (2). Total investment using Ewens, Peters, and Wang's (2020) estimate of the fraction of SG&A invested in organizational capital (scaled by the denominator of lagged EPW q), lagged EPW q (q^{epw}), and total cash flow (c^{tot}) using the same estimate of the fraction of SG&A invested in organizational capital (scaled by the denominator of lagged EPW q) are used in Columns (3) and (4). Block bootstrapped standard errors treating each firm as one block are computed based on 500 replications and are reported in parentheses.

	(1)	(2)	(3)	(4)
	Total investment			
q^{pat}	0.0630	-0.0301		
	(0.0018)	(0.0046)		
c^{tot}	0.6292	0.4783		
	(0.0112)	(0.0291)		
$(q^{pat})^2$		0.0492		
		(0.0056)		
$(q^{pat})^3$		-0.0040		
		(0.0008)		
q^{epw}			0.0804	-0.0061
			(0.0024)	(0.0056)
c^{tot}			0.3279	0.2448
			(0.0173)	(0.0225)
$(q^{epw})^2$				0.0644
				(0.0042)
$(q^{epw})^3$				-0.0102
				(0.0007)
Log-likelihood	-104735.87	-103408.08	-43086.53	-42672.65
LR test statistic		2655.58		827.76
Observations	58,796	58,796	58,796	58,796

Table 5. Investment-q relation in different periods

This table reports investment-q relation in different periods estimated by measurement-error-robust MLE using the Hu and Schennach (2008) IV estimator. Total investment, lagged total q (q^{tot}), and contemporaneous total cash flow (c^{tot}) are employed. Block bootstrapped standard errors treating each firm as one block are computed based on 500 replications and are reported in parentheses.

	(1)	(2)	(3)	(4)
Panel A: Before 2000	1983 – 1991		1992 – 1999	
q^{tot}	0.0901	-0.0328	0.0648	-0.0167
	(0.0054)	(0.0583)	(0.0031)	(0.0077)
c^{tot}	0.5149	0.5014	0.4653	0.4306
	(0.0277)	(0.0270)	(0.0224)	(0.0253)
$(q^{tot})^2$		0.1239		0.0569
		(0.0489)		(0.0054)
$(q^{tot})^3$		-0.0314		-0.0094
		(0.0114)		(0.0008)
Log-likelihood	-3575.47	-3546.82	-13148.40	-13057.39
LR test statistic		57.30		182.02
Observations	10,407	10,407	16,988	16,988
Panel B: After 2000	2000 – 2008		2009 – 2017	
q^{tot}	0.0652	0.0225	0.0867	-0.0081
	(0.0033)	(0.0071)	(0.0038)	(0.1899)
c^{tot}	0.2452	0.2026	0.1283	0.1080
	(0.0232)	(0.0326)	(0.0207)	(0.0884)
$(q^{tot})^2$		0.0289		0.0764
		(0.0051)		(0.1165)
$(q^{tot})^3$		-0.0043		-0.0152
		(0.0008)		(0.0226)
Log-likelihood	-11513.63	-11470.89	-3459.75	-3376.47
LR test statistic		85.48		166.56
Observations	17,774	17,774	13,627	13,627

Table 6. Classic investment model estimated by the cumulant estimator

This table reports the classic investment model estimated by OLS and the Erickson, Jiang, and Whited (2014) cumulant estimator. Total investment, lagged total q (q^{tot}) in Columns (1) and (2), the analyst-based measure of lagged total q ($\hat{q}^{tot}$) in Columns (3) and (4), and contemporaneous total cash flow (c^{tot}) are employed. The original data are within-year transformed to accommodate year fixed effects. ρ^2 is the within-year R^2 from the hypothetical regression of investment on the true q and cash flow. τ^2 is the within-year R^2 from the hypothetical regression of a q proxy on the true q. Standard errors clustered by firm are reported in parentheses.

	(1)	(2)	(3)	(4)
Total investment	OLS	Cumulant	OLS	Cumulant
q^{tot}	0.0203	0.0820		
	(0.0010)	(0.0023)		
$\hat{q}^{tot}$			0.0186	0.0697
			(0.0007)	(0.0017)
c^{tot}	0.4285	0.0313	0.3833	-0.1004
	(0.0092)	(0.0220)	(0.0095)	(0.0210)
Adjusted R^2	0.4316		0.4395	
ρ^2		0.5130		0.5329
		(0.0123)		(0.0114)
τ^2		0.4853		0.5669
		(0.0167)		(0.0132)
Observations	58,796	58,796	58,796	58,796

Table 7. Classic investment model estimated by the cumulant estimator in different periods

This table reports the classic investment model estimated by OLS and the Erickson, Jiang, and Whited (2014) cumulant estimator in different periods. Total investment, lagged total q (q^{tot}), and contemporaneous total cash flow (c^{tot}) are employed. The original data are within-year transformed to accommodate year fixed effects. ρ^2 is the within-year R^2 from the hypothetical regression of investment on the true q and cash flow. τ^2 is the within-year R^2 from the hypothetical regression of a q proxy on the true q. Standard errors clustered by firm are reported in parentheses.

	(1)	(2)	(3)	(4)
	OLS	Cumulant	OLS	Cumulant
Panel A: Before 2000	1983 – 1991		1992 – 1999	
q^{tot}	0.0183	0.1370	0.0231	0.0905
	(0.0026)	(0.0174)	(0.0016)	(0.0040)
c^{tot}	0.5764	0.0520	0.4739	0.0401
	(0.0213)	(0.0848)	(0.0145)	(0.0340)
Adjusted R^2	0.4386		0.4298	
ρ^2		0.5445		0.5547
		(0.0326)		(0.0182)
τ^2		0.3717		0.5010
		(0.0355)		(0.0209)
Observations	10,407	10,407	16,988	16,988
Panel B: After 2000	2000 – 2008		2009 – 2017	
q^{tot}	0.0222	0.0671	0.0171	0.0810
	(0.0013)	(0.0021)	(0.0022)	(0.0062)
c^{tot}	0.3178	-0.0169	0.3274	-0.1117
	(0.0136)	(0.0255)	(0.0187)	(0.0541)
Adjusted R^2	0.3936		0.3075	
ρ^2		0.5341		0.4316
		(0.0189)		(0.0305)
τ^2		0.5449		0.4860
		(0.0236)		(0.0341)
Observations	17,774	17,774	13,627	13,627

Table 8. Net payout-Tobin's q relation

This table reports net payout-Tobin's q relation during the entire and subperiods estimated by OLS. Results are from regressions of net payout on lagged total q (q^{tot}), contemporaneous total cash flow (c^{tot}), and year fixed effects. Standard errors clustered by firm are reported in parentheses.

	(1)	(2)	(3)	(4)	(5)
Net payout	Full	1983–1991	1992–1999	2000–2008	2009–2017
q^{tot}	0.0044	0.0018	0.0023	0.0048	0.0083
	(0.0004)	(0.0008)	(0.0005)	(0.0006)	(0.0012)
c^{tot}	0.0410	0.0209	0.0158	0.0598	0.0991
	(0.0032)	(0.0061)	(0.0042)	(0.0052)	(0.0098)
Adjusted R^2	0.0817	0.0164	0.0237	0.1060	0.1478
Observations	58,796	10,407	16,988	17,774	13,627

Appendix: Identification

We consider that the joint density of i and q^A, q^M, c, q^{IV} admits a bounded density with respect to the product measure of some dominating measure μ defined on $\mathcal{I}$ and the Lebesgue measure on $\mathcal{Q}^{\mathcal{A}} \times \mathcal{Q}^{\mathcal{M}} \times \mathcal{C} \times \mathcal{Q}^{\mathcal{IV}}$. We assume that all marginal and conditional densities are also bounded. We consider all the random variables to be jointly continuously distributed in this treatment. For the identification of the density of our interest, $f_{i|q^M c}(i|q^M, c)$, we make similar assumptions like those in Hu and Schennach (2008) and Song (2015).

Assumption 1. *(i)* $f_{i|q^A q^M c\, q^{IV}}(i|q^A, q^M, c, q^{IV}) = f_{i|q^M c}(i|q^M, c)$ *for all* $(i, q^A, q^M, c, q^{IV}) \in \mathcal{I} \times \mathcal{Q}^{\mathcal{A}} \times \mathcal{Q}^{\mathcal{M}} \times \mathcal{C} \times \mathcal{Q}^{\mathcal{IV}}$ *and (ii)* $f_{q^A|q^M c\, q^{IV}}(q^A|q^M, c, q^{IV}) = f_{q^A|q^M c}(q^A|q^M, c)$ *for all* $(q^A, q^M, c, q^{IV}) \in \mathcal{Q}^{\mathcal{A}} \times \mathcal{Q}^{\mathcal{M}} \times \mathcal{C} \times \mathcal{Q}^{\mathcal{IV}}$.

Assumption 1 (i) specifies that the observed regressor q^A and the instrument q^{IV} do not provide any more information about the dependent variable i than the true unobserved regressor q^M and the additional observed regressor c already provide. This is satisfied when measurement errors in q^A and q^{IV} do not immediately affect unobserved causes of i. Similarly, Assumption 1 (ii) indicates that q^{IV} does not provide any further information about q^A given q^M and c, which is satisfied when measurement error in q^A is independent of q^{IV} given q^M and c. These assumptions can be interpreted as exclusion restrictions in the standard IV approach.

Let a and b denote random variables with respective supports $\mathcal{A}$ and $\mathcal{B}$. Given two corresponding spaces $\mathcal{G}(\mathcal{A})$ and $\mathcal{G}(\mathcal{B})$ of functions with domains $\mathcal{A}$ and $\mathcal{B}$, respectively, let $L_{b|a}$ denote the operator mapping $\mathrm{g} \in \mathcal{G}(\mathcal{A})$ to $L_{b|a}\mathrm{g} \in \mathcal{G}(\mathcal{B})$ defined by

$$[L_{b|a}\mathrm{g}](b) \equiv \int_{\mathcal{A}} f_{b|a}(b|a)\, \mathrm{g}(a)\, da.^8$$

Assumption 2. *The operators* $L_{q^A|q^M c}$ *and* $L_{q^{IV}|q^A c}$ *are injective.*

[8] We refer to Definition 1 in Hu and Schennach (2008).

The space $\mathcal{G}(\mathcal{A})$ upon which the operator $L_{b|a}$ acts must be sufficiently large so that the density $f_{b|a}(b|a)$ can be sampled everywhere and thus be uniquely determined by the operator. Assumption 2 imposes restrictions on the relationships between q^A, q^M, c, and q^{IV}. An operator $L_{b|a}$ is injective if there is enough variation in the density of b for different values of a. In this sense, Assumption 2 implies that given c, q^A has enough information about q^M, and q^{IV} has enough information about q^A. This condition is related to the rank condition in the standard IV approach and easily holds as long as the amounts of measurement errors are reasonable.

Assumption 3. *For any $c \in \mathcal{C}$ and any q_1^M, $q_2^M \in \mathcal{Q}^{\mathcal{M}}$, the set $\{i : f_{i|q^M c}(i|q_1^M, c) \neq f_{i|q^M c}(i|q_2^M, c)\}$ has a positive probability whenever $q_1^M \neq q_2^M$.*

Assumption 4. *For any given $c \in \mathcal{C}$, there exists a known functional M such that $M[f_{q^A|q^M c}(\cdot|q^M, c)] = q^M$ for all $q^M \in \mathcal{Q}^{\mathcal{M}}$.*

Assumptions 3 and 4 ensure a unique decomposition of an integral operator associated with the joint density of the observables. Assumption 3 is only violated if the distribution of i conditional on q^M and c is identical at different values of q^M, so the presence of conditional heteroskedasticity or monotonicity of i in q^M conditional on c is sufficient to satisfy the assumption. Assumption 4 places a restriction on some measure of the location of a density. M is a general functional mapping a density to a real number. This assumption allows for measurement error in the model to be either classical or nonclassical because M can take any form. To deal with the case of nonclassical measurement error, Hu and Schennach (2008) exploits the observation that even though measurement error may not have zero mean conditional on the true regressors, some other measures of location (e.g., mode or median) could still be zero.[9] Assumption 4 is invoked by this observation.

[9] For example, consider M that defines the mode or the median of a density: $M[f] = \underset{x \in \mathcal{X}}{\arg\max} f(x)$, $M[f] = \inf\{x^* \in \mathcal{X}^* : \int 1(x \leq x^*) f(x) dx \geq 0.5\}$ where x^* is the unobserved true regressor and x is the error-contaminated counterpart. These are two examples of M that cover nonclassical measurement error in x of various forms.

Theorem 1. *Under Assumptions 1–4, given the observed density* $f_{i\,q^A|c\,q^{IV}}(i, q^A|c, q^{IV})$*, the equation*

$$f_{i\,q^A|c\,q^{IV}}(i, q^A|c, q^{IV}) = \int_{\mathcal{Q^M}} f_{i|q^M c}(i|q^M, c) f_{q^A|q^M c}(q^A|q^M, c) f_{q^M|q^{IV} c}(q^M|q^{IV}, c)\, dq^M \qquad (8')$$

admits a unique solution $(f_{i|q^M c}, f_{q^A|q^M c}, f_{q^M|q^{IV} c})$ *for all* $i \in \mathcal{I}$*,* $q^A \in \mathcal{Q^A}$*,* $c \in \mathcal{C}$*,* $q^{IV} \in \mathcal{Q^{IV}}$*.*

Assumption 1 facilitates the operation of integration in Equation (8′) that relates the joint density of the observables to the joint densities of the unobservable variable q^M. Specifically, the identification of the density of interest $f_{i|q^M c}$ can be achieved through the eigenvalue-eigenfunction decomposition of an integral operator associated with the joint density of the observables. Assumptions 3 and 4 ensure uniqueness of this decomposition.