

A Theory for Measures of Tail Risk

Abstract:

The notion of "tail risk" has been a crucial consideration in modern risk management and financial regulation, as very well documented in the recent regulatory documents. To achieve a comprehensive understanding of the tail risk, we carry out an axiomatic study for risk measures which quantify the tail risk, that is, the behavior of a risk beyond a certain quantile. Such risk measures are referred to as tail risk measures in this paper. The two popular classes of regulatory risk measures in banking and insurance, the Value-at-Risk (VaR) and the Expected Shortfall (ES), are prominent, yet elementary, examples of tail risk measures. We establish a connection between a tail risk measure and a corresponding law-invariant risk measure, called its generator, and investigate their joint properties. A tail risk measure inherits many properties from its generator, but not subadditivity or convexity; nevertheless, a tail risk measure is coherent if and only if its generator is coherent. We explore further relevant issues on tail risk measures, such as bounds, distortion risk measures, risk aggregation, elicibility, and dual representations. In particular, there is no elicitable, tail convex risk measure rather than the essential supremum, and under a continuity condition, the only elicitable and positively homogeneous monetary tail risk measures are the VaRs.