THE ILLIQUIDITY FOG

Anjan V. Thakor
Olin Business School
Washington University in St. Louis
thakor@wustl.edu

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Abstract

This paper develops a theory of how heterogeneity in bank capital choices due to differences in beliefs about the likelihood of a future crisis state leads to partially frozen credit markets in a crisis—some banks have continued access to funding liquidity but others do not, creating an "illiquidity fog". Interbank trading in legacy assets allows some frozen banks to sell assets to obtain funding. Consequently, there is a reallocation of access to market funding from low-capital banks to high-capital banks. There are strategic complementarities in the capital choices of high-capital and low-capital banks, leading to a "capital structure contagion".

JEL classification: G01, G20, G21

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"Our liquidity is fine. As a matter of fact, it's better than fine. It's strong."

Kenneth lay, CEO of Enron, October 23, 2001.

I. INTRODUCTION

It is widely believed that the financial stresses that emerged in 2007 due to unexpectedly high mortgage delinquencies eventually led to financial institutions being frozen out of short-term funding markets. Philippon and Skreta (2012) and Tirole (2012) have proposed that this illiquidity stemmed from adverse selection about legacy assets, and it justified government intervention in these markets. Specifically, they show how the mechanism design for government equity injections and asset purchases interacts with the endogenous participation constraints of institutions that can avail of market financing as an alternative to government funding. A key insight of these papers is that, unlike the standard mechanism-design problem, the firms that choose not to participate in the government's mechanism can access market financing, thereby benefiting from the market rebound that follows the "cleansing" provided by the governmental intervention.

Arrayed against the view that there was a marketwide liquidity freeze that rationalized taxpayer-subsidized government intervention is recent empirical evidence that there was *no* such marketwide freeze. Rather, as financial stresses grew, the short-term funding market reallocated liquidity from banks with lower capital and poor-quality assets to those with higher capital and better assets. Perignon, Thesmar and Vuillemey (2018) use transaction-level data on short-term, unsecured certificates of deposit in the European market to document that there was no marketwide funding freeze during 2008–14, but many banks experienced sudden funding dry-ups. They show that banks with higher capital and better future performance actually *increased* their short-term uninsured funding, whereas those with lower capital and poorer future performance reduced their funding. Similarly, Boyson, Helwege and Jindra (2014) use U.S. data to document that commercial bank funding did not dry up even during the 2007–09 crisis, but the market forced weak (low-capital) banks to borrow less.

How do we reconcile this empirical evidence with the view that funding markets may be frozen—albeit selectively—for some institutions during financial crises? In this paper, I develop a model that addresses this research question. The model also helps to address related questions like: what gives rise to the market-freeze-inducing adverse selection in the first place? That is, apart from an overall deterioration in economic conditions that exacerbates adverse selection, can we identify pre-crisis conditions that contribute to the market freeze during the crisis? An improved understanding of this would inform regulatory policies that may be undertaken well before the crisis that would improve market access to funding during the crisis, lowering the burden on taxpayers.

In this paper, I address these issues by developing a model in which the capital levels chosen by banks determine the probability with which they end up in a market frozen by adverse selection in a future economic downturn. High-capital banks avoid a market freeze even in an economic downturn in which *fundamentals* (i.e., credit quality of legacy assets) decline for *all* banks. However, in this economic state, low-capital banks experience a market freeze, unable to access funding for new (positive-NPV) projects. As in Tirole (2012), lack of pledgeability prevents raising financing by issuing (securitized) claims against the new projects, so all claims to finance these projects must be issued against legacy assets—where the adverse selection problem resides.[1] The reason why a bank's capital level influences its future vulnerability to a market freeze is that it affects its incentives to screen borrowers, and loans to these borrowers become the legacy assets with which banks enter the next period. Higher-capital banks invest more in screening and have better legacy assets on average. The bank's screening signal is informative but noisy, so even some loans screened as creditworthy are not. The actual creditworthiness of loans is revealed privately to each originating bank at an interim future date before the loans mature, and it depends on the initial screening-based identification as well as the realization of a macroeconomic state (boom or bust). On average, loan qualities are better in a boom than in a bust. No bank experiences a freeze in a boom, but in a bust, the low-capital banks experience a freeze because their legacy assets are relatively low quality on average and unable to support financing for the new project.

In contrast to earlier research, I allow for the possibility of "expert buyers" of assets who can evaluate the quality of another bank's legacy assets at a cost.[2] This secondary market for loan sales gives frozen banks an alternative to raising financing for the new project in the market, as well as an alternative to government assistance. Thus, the frozen banks are not drowned by their illiquidity, absent government intervention. Rather, they are in an "illiquidity fog" that can be dispelled through bilateral trades with informed buyers. This feature is of no use in previous models like Tirole (2012) and Philippon and Skreta (2012) because *all* banks are frozen, so none is available as a buyer. Bank heterogeneity, introduced by their initial (endogenous) capital structure choices, leads to a market segmentation with possible gains from trade.

[1] Similar to Myers and Majluf (1984).

[2] This potentially helps banks to avoid fire sales. In the Shleifer and Vishny (2011) model, *all* expert buyers are financially constrained.

This model produces the following key results. First, banks that assign a higher probability to a future crisis state choose higher levels of capital and invest more in screening precision. Consequently, they end up with higher-quality legacy assets when the crisis arrives. Second, there are strategic complementarities in the capital structure choices of banks. As some banks increase their capital in response to a higher perceived probability of a future crisis state, other banks that believe the probability of a future crisis is lower also increase their own capital levels.[3] That is, there is a sort of "capital structure contagion." Third, consistent with the earlier-cited empirical evidence, the legacy asset resale market leads to an *ex post* reallocation of liquidity from low-capital to high-capital banks. Fourth, not all low-capital banks will be able to sell their assets in a crisis. Thus, after the resale market clears, there is room for welfare-enhancing government intervention. Finally, I rely on recent research on welfare analysis with heterogeneous beliefs (Brunnermeier, Simsek and Xiong (2014)) to provide some additional results. One of these is that the social planner can never improve welfare by adopting a belief more optimistic than that of the pessimist. Another result shows the kind of *ex post* intervention the government could engage in that would improve welfare but not cost the government anything.[4]

The analysis has the policy implication that increasing capital levels in banks not only reduces insolvency risk when the capital levels are raised, but it may also reduce the risk of a future market freeze due to adverse selection. In light of the Malherbe (2014) result that imposing liquidity requirements on financial institutions may exacerbate future liquidity dry-ups,

[3] Our result obtains even though there are no direct risk spillover effects or other forms of contagion across banks. Such effects, for which there is empirical evidence, are likely to strengthen our strategic complementarity result. Cesa-Bianchi, Eguren Martin and Thwaites (2018) provide evidence that *foreign* credit growth affects the probability of a domestic crisis, even controlling for domestic credit growth, suggesting cross-border spillover effects.

[4] Nonetheless, there is a residual distortion relative to the first best.

addressing potential illiquidity via capital regulation may be better than doing so via liquidity regulation. This runs counter to the assumptions underlying the adoption of liquidity requirements in Basel III.

This paper is related to various strands of the literature. The closest relationship is to the literature on the potential effects of government intervention in markets frozen by adverse selection. Examples are Jorge and Kahn (2014),[5] Philippon and Skreta (2012), Tirole (2012), and Chiu and Koeppl (2015). While the first three papers focus on the role of post-market-freeze government intervention in static settings, Chiu and Koeppl (2015) examine trading dynamics and optimal government intervention when finding a counterparty takes time. The paper establishes conditions under which the government optimally delays its costly intervention to buy up lemons.[6] Camargo and Lester (2014) show how a dynamic, decentralized market suffering from adverse selection recovers *endogenously* over time. This paper differs from these papers in two important respects. One is the introduction of informed buyers of legacy assets who provide an alternative to market financing, and the other is the endogenous bank equity choice. These features lead to an interaction between a tool typically used for solvency regulation—capital requirements—with illiquidity risk. Further, the welfare analysis focuses on a possible welfare-enhancing intervention that is costless to the taxpayers, in contrast to the earlier research in which it is costly.

Another strand of the literature this paper is related to is regulatory recapitalization of banks in an environment with moral hazard and adverse selection. Relevant papers are Acharya,

[5] Jorge and Kahn (2014) also examine policies that promote *ex ante* insurance against liquidity shocks.

[6] There are also papers about pre-market-freeze government initiatives. For example, Madison (2018) examines how monetary policy interventions that replace information-sensitive assets with government bonds or fiat money can improve welfare. Allen, Gale and Carletti (2009) develop a model of interbank trading in a *safe*, long-term asset in which the central bank can implement the constrained-efficient allocation using open market operations, but a market freeze may be a feature of this allocation.

Mehran and Thakor (2016), Acharya and Thakor (2016), Landier and Ueda (2009), Philippon and Schnabl (2013), Aghion, Bolton and Fries (1999), and Plantin (2015). The analysis in this paper does not deal directly with forced regulatory recapitalization, although there are implications for regulatory capital requirements that are discussed.

The third strand is the literature in which future adverse selection is affected by current decisions. In Chari, Shorideh and Zetlin-Jones (2014), current selling decisions convey information that has an impact on future adverse selection. Malherbe (2014) develops a model in which large current cash holdings worsen future adverse selection because a smaller number of future asset sales are due to cash needs. In Plantin's (2009) model, current investment decisions depend on anticipation about future liquidity.[7] None of these papers considers the roles of capital and bilateral transactions with informed buyers. Thus, they do not focus on the reallocation of liquidity from weak to strong banks in markets initially frozen by adverse selection, but which partially unfreeze on their own.

Finally, as mentioned earlier, the paper is also related to the fire-sales literature, pioneered by Allen and Gale (1994) and Shleifer and Vishny (2011). In these models, due to cash-in-the-market pricing constraints (Allen and Gale (1994)) or due to the necessity of selling to non-expert second-best users, the price falls with asset sales. Recent extensions have examined variations of these initial settings. Dow and Han (2017) show that fire sales can occur even when there are well-capitalized *non-expert* buyers. By contrast, in the model here, there are well-capitalized *specialist* buyers. Kurlat (2016) studies the problem of privately-informed sellers in a market in which there is heterogeneity among buyers in the quality of their

[7] The anticipation of future liquidity can affect investment decisions in various ways. For example, Acharya and Naqvi (2018) show how monetary loosening induces banks to increase their "reach for yield" and overinvest in risky assets.

information about the asset. That is, the model focuses on the role of buyer expertise, and conditions are derived under which such economies involve fire sales. An important difference between that paper, in which prior beliefs are homogeneous, is that buyers are exogenously heterogeneous in their information quality, whereas in this paper the segmentation of the market into buyers and sellers arises endogenously due to different capital structure choices driven by heterogeneous beliefs.[8]

The rest of the paper is organized as follows. Section II describes the model. Section III has the analysis. Section IV discusses the policy implications of the analysis. Section V concludes. All proofs are in an Appendix.

II. MODEL

Consider an economy in which all agents are risk neutral and the risk-free rate of interest is zero. There are three dates: t=0, 1, and 2. At t=0, there are numerous banks, each of which is making three decisions: (i) whether to make a loan of I to an applicant, (ii) the mix of deposits and equity with which to finance the loan, and (iii) how much to invest in a screening technology to determine if the loan applicant is creditworthy.

Borrower and Bank Types. Loan applicants can be one of three types: good (G), medium (M) or bad (B). A G borrower needs I to invest in a project that pays off Y for sure at t=2. As an alternative to this project, the borrower could also invest in a private-benefit project that does not generate a cash flow that can be contracted upon, but yields the borrower a private benefit of $\beta \in \mathbb{R}_+$.

[8] The focus of Kurlat's (2016) analysis is entirely different from the focus here. His main focus is a normative analysis of competitive asset markets in which sellers are privately informed, but unlike Akerlof (1970), buyers are not all equally informationally disadvantaged.

A B borrower also seeks a loan of I at t=0. But this borrower's project payoff at t=2 from the bank loan is always 0. This borrower too has a private-benefit project that yields $\beta > 0$, so this borrower will always invest in it if it receives a bank loan. An M borrower's payoff at t=2 depends on the realization of a macroeconomic state, s, at t=1, and $s \in \{b, r\}$, where "b" stands for "boom" and "r" stands for recession. If s=b, then the M borrower becomes G. But, if s=r, then M's borrower becomes B. It is common knowledge that $\Pr(s = b) = \theta \in (0,1)$.

These three types of borrowers fall into two pools: P_G and P_B. The P_G pool consists of only type G and type M borrowers, whereas the P_B pool consists of only type B borrowers. Each borrower knows which pool it is in, but no one else does. Within the P_G pool, borrowers do not know whether they are type G or type M. Conditional on the borrower being in the P_G pool, the common prior belief is that $\Pr(G \mid P_G) = g \in (0.5,1)$.

Banks can be one of two types: normal (N) and lemons (L). No one can distinguish between these banks at t=0, and even the banks themselves do not know their types at t=0. The common prior belief is that the probability is $v \in (0,1)$ that the bank is type N. If the bank is type N, then it has the P_G borrower pool. If the bank is type L, then it has the P_B borrower pool.

Thus, the borrower's maximum pledgeable cash flow is X, given as a solution to:

$v[Y - X]\big[g + [1-g]\theta\big] = \beta$, so

$$X = Y - \frac{\beta}{v\big[g + [1-g]\theta\big]} \tag{1}$$

As in Holmstrom and Tirole (1997), it will be assumed that the borrower's repayment obligation on the loan will be set by the bank at X, conditional on the borrower getting the loan.

Deposits are uninsured, but assumed to be cheaper than equity for the bank. This is a standard assumption and can be justified in many ways, including taxes, implicit safety-net protection for uninsured bank creditors, etc. The additional cost of equity is captured by assuming there is a cost of equity capital of $\psi(E)$, with $\psi' > 0$, $\psi'' > 0$, $\lim_{E \to 0} \psi' = 0$. That is, while deposits are priced so that the expected value of the repayment to depositors equals the amount of deposits raised, with equity there is a cost of $\psi(E)$, which represents the cost to the bank's insiders of providing E in equity. The bank is run to maximize the wealth of these insiders. Clearly, in the first best, this means that the bank is all deposit-financed.

Bank Screening: The bank can screen a loan applicant to determine whether the applicant is creditworthy. The precision of the screening is η, where the signal produced by the screening is $\phi \in \{G, \text{not } G\}$ and

$$\Pr(\phi = G \mid G) = \eta = \Pr(\phi = \text{not } G \mid B \text{ or } M) \tag{2}$$

where $\eta \in [0.5, \bar{\eta}] \subset [0.5, 1]$. the cost of screening is $C(\eta)$, with

$$C' > 0,\ C'' > 0, C''' > 0 \left(\text{sufficiently large}\right),\quad \lim_{\eta \to 0.5} C' = 0,\ \lim_{\eta \to \bar{\eta}} C' = \infty.$$

This is a one-time investment in screening and it is privately observed only by the bank making the investment. Once it is made, screening at a future date with precision η involves no additional cost. Thus, the signal identifies whether a loan is good or not, but that is identified by ϕ as not *G, M, or B* means that the bank's uncertainty about its own type is not resolved by observing φ.

Deposit Contract: The bank raises deposits $D \in [0, I]$ at t=0. If the depositors are paid off at t=1, the contractual repayment is D. If the bank chooses to repay at t=2, the repayment is $\bar{D}(s)$, which is a function of the macroeconomic state s at t=1 and will be endogenously solved to yield depositors their required expected return of zero. A covenant in the deposit contract is that the bank must pay off depositors at t=1 if it sells is legacy loan at that time.

Heterogeneity of Beliefs: There are two groups of banks, distinguished by the value assigned to θ. One group believes that $\theta = \theta_1$ and the other group believes that $\theta = \theta_2$, where $1 > \theta_1 > \theta_2 > 0$, This assumption leads directly to banks choosing different capital structures, which is heterogeneity that is needed for the analysis.

Events at *t*=1: At t=1, the macroeconomic state s is realized and becomes common knowledge. Each bank can decide whether or not to pay off its date-0 depositors. Each bank also decides whether to raise and additional I at t=1 to fund a new asset that pays off R at t=2. As in Tirole (2012), it is assumed that the pledgeability of the cash flow generated at t=2 by this new asset is so limited that the financing I cannot be raised at t=1 if only this new asset is available to repay financiers. For simplicity, the extreme assumption is made that the entire payoff of the new asset, R, is not pledgeable[9], where $R > I$. Thus, the financing for this asset is made possible by giving the financiers at t=1 access to the payoff generated at t=2 by the legacy loan made at t=0.

At t=1, each bank's type (N or L) becomes publicly known. Each N-type bank also *privately* observes whether the loan it made at t=0 was to a G or a M borrower and since is realized, the bank knows whether the borrower is G or B. Since all financing raised at t=1 is a claim against the cash flow of this date-0 legacy loan, the bank's private information generates adverse selection.

As an alternative to raising market financing, a bank may decide to sell its legacy loan to another bank. A bank that purchases the loan can avail of its screening technology investment at t=0 to screen the loan it buys at t=1, but locating a seller and completing the purchase transaction involves a cost of $\varphi > 0$.[10] In the analysis, φ is an arbitrarily small number. The price at which

[9] As Tirole (2012) shows, this assumption can be easily relaxed.

[10] It is assumed that the selling bank incurs no cost in selling its legacy loan. Introducing a cost for the selling bank does not affect the analysis.

trading occurs will depend on whether there is a scarcity of buyers or sellers. Since it is unknown *ex ante* which situation will occur, on an expected value basis the gain from trade is split between the buyer and the seller, so the expected trading price is intermediate between the minimum price the buyer is willing to accept and the maximum price the seller is willing to pay.

Figure 1 provides a summary of the events at t=0, 1, and 2.

Figure 1: Events at Different Dates		
t=0	t=1	t=2
• A bank can be either normal (N) or a lemon (L) and no one knows the bank's type. • Each bank has a belief about the macro state s at t=1: a probability $\theta \in \{\theta_1, \theta_2\}$ that s=boom. • Each bank chooses a capital structure (mix of uninsured deposits and equity) to finance a loan. • Each bank decides on a privately-observed investment in screening that determines the precision with which it screens borrowers to determine creditworthiness.	• $s \in \{\text{boom, recession}\}$ is realized. Each bank's type (L or N) becomes publicly known. • Bank can decide whether to pay off deposits raised at t=0 or pay them off at t=1. • If possible, each bank raises financing for a new asset as well for paying off date-0 deposits. • Bank may either purchase another bank's legacy loan (made at t=0) or sell its own legacy loan. A loan purchase requires additional deposit financing to be raised.	• Both the legacy loan and the new asset purchased pay off. • All depositors are paid off, and bank shareholders (insiders) collect the rest.

Restrictions on Exogenous Parameters

In what follows, we will impose restrictions on the exogenous parameters in order to focus on the cases of interest. While the restrictions are described verbally below, the corresponding mathematical expressions are given in the Appendix.

(R1) *Limited Ex Ante Financing:* It is assumed that, due to the presence of *L* banks at *t*=0 the bank can raise enough financing for the loan at *t*=0, but not enough to also finance the new asset at *t*=1.

(R2) *Financing the new asset at t=1 is optimal as long as the cost of financing is not excessive:* This means that not all banks will choose to finance the new asset.

(R3) *The borrower's pledgeable cash flow, X, is large, but not too large:* This simply accommodates the possibility that in some cases the market is willing to finance firms at *t*=1 and in some cases it is not.

(R4) *The surplus from the new asset available at t=1 is sufficiently large but not too large.* The idea is that the surplus from the new asset at t=1 is large enough to provide some banks an incentive to incur a modest adverse-selection cost to raise financing for the new asset, but it is not so large that banks would be willing to incur any cost to raise financing for it.

(R5) *The borrower pool for normal banks is of sufficiently high quality.*

(R6) *A sufficiently high screening precision is available:* This gives the low-capital banks sufficient borrowing capacity to be asset buyers at *t*=1.

III ANALYSIS

The model will be solved (as usual) by analyzing the second period first and then moving to the first period—we start with the events at *t*=1 and then analyze the events at *t*=0. We will show that banks with the belief $\theta = \theta_1$ will choose a low level of equity capital, E_ℓ, and banks with the belief $\theta = \theta_2$ will choose a high level of equity capital, $E_h > E_\ell$. We will also show that banks choosing E_ℓ and E_h will invest different amounts in screening precision, achieving precisions of η_ℓ and η_h respectively, with $\eta_\ell < \eta_h$.

For our analysis of events at *t*=1, we will take these results as given and then prove them in the analysis of events at *t*=0. Given Restriction R1, the bank will raise *I* at *t*=0 for its first-period lending.

A. Events at *t*=1

At *t*=1, each bank has to decide whether to pay off its first-period depositors, whether to raise financing for the new asset, and whether to sell its legacy loan or buy another bank's legacy loan. Since the bank's type becomes known at *t*=1, only the type *N* banks can raise financing.

***Low-Capital Banks*:** Consider first the banks with $E = E_\ell$ and $\eta = \eta_\ell$. Suppose *s*=*b*. In this case, it is common knowledge that all banks' legacy loans are *G*, so given (R-2), each bank can raise financing at $t = t_0$ to pay off its first-period deposits, and finance the new asset.

Lemma 1*: If the macroeconomic state at t=1 is a boom, the first-period depositors will need to be paid D if they are paid off at t=1 and D if they are paid off at t=2. Each low-capital bank chooses to raise sufficient external financing to pay off deposits raised at t=0 and finance the new asset. All external financing raised at t=1 is in the form of deposits.*

The intuition is that the bank's *G* legacy loan provides sufficient borrowing capacity to enable it to raise funding to pay off the date-0 depositors and also finance the new asset which as positive-NPV to the bank's insiders. The financing at *t*=1 is all debt because no screening investment has to be made at that date—the only role of bank equity capital is to provide skin in the game to induce investment in screening.

Lemma 2: *if s=b at t=1, there is no trading among banks for their legacy loans.*

Essentially, in a boom all banks are on an equal footing, so there are no gains from trade. However, trading does involve a cost for the buyer, so the net gain from trade is negative.

Next suppose $s = r.$ Now the probability that the low-capital bank has a *G* loan is given by:

$$\Pr^{\ell}(G) \equiv f_{\ell} = \frac{g\eta_{\ell}}{g\eta_{\ell} + [1-\eta_{\ell}][1-g]} \tag{3}$$

Assume for now that that $f_{\ell} < g^{*};$ this will be verified later.

The next result characterizes the terms of the deposit contract when *s*=*r*.

Lemma 3: *When s=r, first-period depositors need to be paid D if paid off at t=1 and* $D[f_{\ell}]^{-1}$ *if paid off at t=2.*

This lemma is intuitive. If all banks choose to pay off first-period depositors at *t*=1, then the deposits they provide at *t*=0 are riskless, so the repayment is *D*. If all banks choose to delay repayment until *t*=2, then the probability of a *G* loan is $f_{\ell},$ so the repayment probability is $f_{\ell}.$ Note that the equilibrium at *t*=1 must be pooling since by not adopting the same strategy of repayment timing as the banks with *G* loans, the banks with *B* loans would reveal themselves.

Proposition 1*: Assuming* $f_\ell < g^*$, *the market freezes for low-capital banks at t=1 and no financing is available. There is a universally divine sequential equilibrium*[11] *in which no bank raises any financing at t=1, and any bank attempting to raise financing is viewed by investors as having a B loan with probability one.*

There is adverse selection in the market for low-capital banks since the banks with *G* loans are unwilling to raise financing for the new asset at the cost at which it would be available if banks with *B* as well as *G* loans were in the market, whereas the banks with *B* loans are willing to raise financing at those terms.

It has been assumed thus far that there will be no purchases of legacy assets of low-capital banks by other low-capital banks. It will be verified next that, as long as there are enough high-capital banks, the buyers of legacy loans will always be the high-capital banks.

High Capital Banks**:** When *s*=*b*, the analysis for these banks mirrors that for the low-capital banks that was done previously. So now consider *s*=*r*. The probability is that a high-capital bank has a G loan given by

$$\Pr^h(G) = \frac{\eta_h g}{\eta_h g + (1-\eta_h)(1-g)} = f_h \tag{4}$$

Lemma 4**:** *There will be no sales and purchases of legacy loans among high-capital banks.*

The basic idea is that even in a recession, the high-capital banks are able to raise financing to pay off legacy depositors and invest in the new asset. Selling a loan to another high-capital bank to raise financing is inefficient if the bank knows it has a *G* loan because the maximum price it can

[11] See Banks and Sobel (1987).

get is less than the true value of the loan. Thus, only banks with *B* loans are willing to sell legacy loans, so the market breaks down.

Lemma 5: *Assuming both high-capital and low-capital banks can raise financing for purchasing a legacy loan and investing in the new asset, the maximum price a high-capital bank can pay for the legacy loan of a low-capital bank exceeds the maximum price a low-capital bank can pay for that loan when s=r.*

The intuition is that the screening precision of a high-capital bank is higher (we will verify this later), so conditional on being willing to purchase the loan, it has greater confidence that the loan it has identified as good is indeed good. So it is willing to pay a higher price. However, as Proposition 1 showed, low-capital banks will simply be unable to raise financing.

If a high-capital bank wants to purchase a legacy loan from a low-capital bank at *t*=1, it will screen the bank's loan. The bank's screening precision is η_h and it takes f_ℓ (see (4)) as its prior belief that the loan is good. So, conditional on $\phi = G,$ its posterior belief that the loan is *G* is

$$\Pr\left(G \mid \phi = G\right) \equiv \hat{f}_h^r = \frac{\eta_h f_\ell}{\eta_h f_\ell + \left[1-\eta_h\right]\left[1-f_\ell\right]} \tag{5}$$

Let $P_{\min}$ be the minimum price that the seller of a *G* loan would be willing to accept at *t*=1. It is the solution to

$$\left[X - D_\ell \left\{f_\ell\right\}^{-1}\right] = P_{\min} - D_\ell - I + R \tag{6}$$

The left-hand side (LHS) of (6) is the bank's expected payoff if it simply holds its legacy loan until *t*=2 and does not finance the new asset. The right-hand side (RHS) is the bank's payoff if it sells the legacy loan for $P_{\min},$ pays off first-period depositors and invests *I* in the new asset; it is assumed that the selling price exceeds $D_\ell + I.$ Thus,

$$P_{\min} = \left[X - D_{\ell} \left\{ f_{\ell} \right\}^{-1} \right] + D_{\ell} + I - R \tag{7}$$

The maximum price that a high-capital bank is willing to pay is:

$$P_{\max}^{1} = \hat{f}_{h}^{r} X - \varphi \tag{8}$$

which is the maximum price the bank would be willing to pay if it could finance the legacy loan purchase, or

$$P_{\max}^{2} = f_{h} \, \hat{f}_{h}^{r} X - \varphi \tag{9}$$

if its repayment obligations on all the financing raised at t=1 could be covered in full only if both its legacy loan and the purchased legacy loan pay off.

If there are more buying banks than selling banks, then $P=P_{min}$. And if there are more selling banks than buying banks, then $P = P_{\max}$.[12] The number of buyers and the number of sellers at t=1 depends on how many banks believed $\theta = \theta_1$ and how many believed $\theta = \theta_2$ at $t = 0$. These are taken as exogenous. Let λ be the probability that $P = P_{\max}$ and $1 - \lambda$ the probability that $P = P_{\min}$. This price should satisfy the following constraint:

$$P \geq I + D_{\ell} \tag{10}$$

The reason for this constraint is that if P is not at least as great as $I + D_{\ell}$, the seller cannot pay off legacy depositors and also invest in the new asset at t=1. By the covenant in the legacy deposit contract that these deposits must be redeemed at t=1 if the bank raises new financing at that time, the bank is first forced to pay off legacy depositors D_{ℓ}. Thus, if the price $P < D_{\ell} + I$, it is pointless for the bank to sell its asset. We now have:

Lemma 6: *Given a sufficiently high investment in screening by the high-capital banks, when s=b, low-capital banks will attempt to sell their legacy loans to high-capital banks, and the*

[12] Whether $P_{\max} = P_{\max}^{1}$ or $P_{\max}^{2}$ will be clarified in the next result.

transaction will occur in each case conditional on the high-capital bank's screening signal $\phi = G.$ *In this case,* $P_{\max} = P_{\max}^2 > P_{\min} \geq D_\ell + I.$

The reason why the market freezes for low-capital banks and not for high-capital banks is that the former have lower screening precision and hence a worse legacy loan portfolio on average. Selling its legacy loan to a high-capital bank offers the low-capital bank an opportunity to overcome the market freeze and use the proceeds to pay off its legacy deposits and buy the new asset The low-capital banks are sellers because they cannot raise financing, whereas the better asset quality of the high-capital banks—due to their superior screening— permits them to raise financing to purchase the loans of other banks.

We now turn to the high-capital bank's financing of the legacy loan purchase. We assume that X is not large enough to repay the bank's legacy deposits and the additional borrowing at t=1 to purchase the new asset as well as the low-capital bank's legacy loan (see R1). This means that when the high-capital bank purchases the legacy loan of a low-capital bank, the shareholders of the high-capital bank can get a positive payoff only in the state in which both the purchased loan and its own legacy loan pay off.

B. Events at *t*=0:

We now turn to the bank's choice of equity capital E and its screening intensity η at t=0, conditional on its belief about θ. We begin by considering the low-capital banks that assume they will be potential sellers of legacy loans when $s = r.$ Then the high-capital banks will be analyzed, taking as a given that these banks view themselves as potential buyers of legacy loans at t=1. Then it will be verified that these banks will find it optimal to pursue the stipulated policies.

Banks that Believe θ=θ₁:

When $s = r,$ let τ be the probability that a bank that has chosen a low capital will be able to sell its legacy loan. Then

$$\tau = \begin{Bmatrix} \Pr(\text{bank has a } G \text{ loan})\Pr(\text{high-capital bank's signal is } G \mid G) \\ +\Pr(\text{has a } B \text{ loan})\Pr(\text{high capital bank's signal is } G \mid B) \end{Bmatrix}$$

so, $\tau = f_\ell \eta_h + [1 - f_\ell][1 - \eta_h]$ (11)

Assuming the bank will be a potential seller of its legacy loan, the low-capital bank's problem can be written as:

$$\Omega \equiv \begin{Bmatrix} \theta_1 [X - D_\ell - I] \\ +[1-\theta_1]\left\{\tau\left[\bar{P} - D_\ell + R - I\right] + (1-\tau) f_\ell \left[X - D_\ell \{f_\ell\}^{-1}\right]\right\} \\ -E - \psi(E) - C(\eta) \end{Bmatrix} \tag{12}$$

subject to

$$D_\ell + E = I \tag{13}$$

and (9). Here, $\bar{P}$ is the expected value of *P*:

$$\bar{P} = \lambda P_{\max} + [1-\lambda] P_{\min} q \tag{14}$$

where $q \in (0,1)$ is the probability that the bank can sell its loan when there are more sellers than buyers.

Lemma 7: *Taking E as given, the first-order and second-order conditions for the bank's optimal choice of* $\eta,$ *call it* $\eta_\ell^*,$ *are satisfied. Moreover,* $\frac{d\eta_\ell^*}{dE} > 0.$ E_ℓ

This leads to the next result.

Lemma 8: *Both* $P_{\min}$ and $P_{\max}$ *are strictly increasing in the low-capital bank's capital* E_ℓ*. Thus,* $\bar{P}$ *is also increasing in* E_ℓ*. Moreover,* $\bar{P}$ *is concave in* E_ℓ*.*

The reason why $\bar{P}$ is increasing in E_ℓ is that $P_{\max}$ and $P_{\min}$ both are increasing in E_ℓ. When E_ℓ is higher, the low-capital bank invests more in screening precision, increasing the probability that the legacy loan being purchased is a *G* loan. This is why both $P_{\max}$ and $P_{\min}$ increase as E_ℓ increases. Next we have,

Proposition 2: *There is a unique interior optimal solution for the amount of equity capital chosen by the bank,* E_ℓ^*, *and* $dE_\ell^* / d\theta_1 < 0.$

The intuition for why the bank's equity capital is decreasing in θ_1 is that it is only in the recession macro state that capital helps the bank. Thus, as the probability of that state declines (as θ_1 increases), the bank keeps less capital. Thus, if θ_1 is low enough, the bank that believes $\theta = \theta_1$ will choose an E_ℓ low enough to ensure a low enough η_ℓ to lead to $f_\ell < g^*$, as previously assumed.

Banks that Believe θ=θ₂:

We start with the assumption that these banks are still buyers. Now for an expected price $\bar{P}$ paid to purchase the legacy loan of a low-capital bank, let R_p be the expected repayment obligation of the bank.

The buying bank's objective function is

$$
\begin{aligned}
\Omega_h = \; & \theta_2 \left\{ \left[X - D_o(h) - I \right] + R \right\} \\
& + [1-\theta_2] \begin{Bmatrix} \tau[f_h][\hat{f}_h^r]\left[2X - \varphi - R_p - D_0(h)\{f_h\}^{-1} - I\{f_h\}^{-1}\right] \\ +R + [1-\tau] f_h \left[X - D_o(h)\{f_h\}^{-1} - I\{f_h\}^{-1} \right] \end{Bmatrix} \\
& + \left[I - D_o(h) \right] - \psi(E) - C(\eta)
\end{aligned} \tag{15}
$$

where R_p is the bank's repayment obligation on its borrowing to purchase a low-capital bank's legacy loan. It is assumed, without loss of generality, that the borrowing to pay off legacy deposits and to purchase the new asset with payoff *R* is senior to the borrowing to buy another bank's legacy asset. Recall that the bank is able to fully repay its junior-most creditors only when both its own legacy loan the purchased legacy loan pay off.

So if the expected price paid for another bank's legacy loan is $\overline{\overline{P}}$ and the bank's expected borrowing to finance the purchase is $\overline{\overline{P}},$ we have:

$$
\overline{\overline{P}} = \left[f_h \hat{f}_h^r \right] R_p + \left[\hat{f}_h^r + f_h - 2\hat{f}_h^r f_h \right] \left[X - \frac{[D_0(h) + I]}{f_h} \right] \tag{16}
$$

where $\hat{q} \in (0,1)$ is the probability that the buyer can buy the loan when there are more buyers than sellers.

$$
\overline{\overline{P}} = \hat{q}\lambda P_{\max} + [1-\lambda] P_{\min} \tag{17}
$$

Rearranging terms yields:

$$
R_p = \frac{\overline{\overline{P}}}{z_1} - \left[z_2 z_2^{-1} \right] \left[f_h X - D_0(h) - I \right] \tag{18}
$$

where

$$
z_1 \equiv f_h \hat{f}_h^r \tag{19}
$$

$$
\begin{aligned}
z_2 &\equiv \hat{f}_h^r + f_h - 2\hat{f}_h^r f_h \\
&= \hat{f}_h^r + f_h - 2z_1
\end{aligned} \tag{20}
$$

$$
z_3 \equiv z_1 f_h \tag{21}
$$

Lemma 9: *$\hat{f}_h^r f_h$ is concave in η_h.*

Lemma 10: *There is an interior optimal choice of screening precision* η_h^* *that satisfies the first-order and second-order conditions for optimality. Moreover,* $d\eta_h^* / dE_h > 0.$

The intuition for the impact of bank capital on its screening precision is that when the bank has more skin in the game, it is willing to invest more in screening.[13] Our next result is about the bank's optimal choice of capital.

Proposition 3: *There is a unique interior optimal solution for the amount of equity capital chosen by the bank,* $E_h^*,$ *and* $dE_h^* / d\theta_2 < 0.$

The intuition for this result is similar to that for Proposition 1 for low-capital banks.

C. Verifying Choices of Banks to be Buyers or Sellers of Legacy Assets

The utility of the high-capital (buyer) bank, given its equilibrium choices, is:

$$\begin{aligned}\Omega_h^* = \;& \theta\left\{\left[X - D_0(h) - I\right] + R\right\} + [1-\theta]\left\{\tau f_h \hat{f}_h^r H_1 + R + [1-\tau] f_h H_2\right\} \\ & - E_h^* - \theta\psi\left(E_h^*\right) - C\left(\eta_h^*\right)\end{aligned} \tag{22}$$

where

$$H_1 \equiv 2X - \varphi - R_p - D_0(h)\left[f_h\right]^{-1} - I\left[f_h\right]^{-2} \tag{23}$$

$$H_2 \equiv X - D_0(h)\left[f_h\right]^{-1} - I\left[f_h\right]^{-1} \tag{24}$$

Similarly, the low-capital (seller) bank's utility is:

[13] Empirical evidence consistent with this has been provided by Purnanandam (2011).

$$\Omega_\ell^* = \theta\left\{\left[X - D_0(\ell) - I\right] + R\right\} + [1-\theta]\begin{Bmatrix}\tau\left[P - D_0(\ell) + R - I\right] \\ +[1-\tau]\, f_\ell\left[X - \bar{D}_0(\ell)\right]\end{Bmatrix}$$
$$-E_\ell^* - \psi\left(E_\ell^*\right) - C\left(\eta_\ell^*\right) \tag{25}$$

The following result indicates how a bank makes its choice about whether to be a low-capital or a high-capital bank.

Proposition 4: *There exists a* $\theta^* \in (0,1)$ *such that a bank with* $\theta_2 \in (0,\theta^*)$ *will choose to have high capital and a bank with* $\theta_1 \in (\theta^*,1)$ *will choose to have low capital. For* θ_2 *low enough, the high-capital bank chooses a sufficiently high screening precision to enable it to raise 2I.*

In light of the earlier analyses, this result is intuitive. When a bank believes θ is high, it chooses low capital, because the value of capital in inducing higher screening precision is low. This validates the assumption maintained throughout the previous analysis that the banks believe $\theta = \theta_1,$ choose low capital and the banks that believe $\theta = \theta_2$ will choose high capital.

IV. Policy Implications

The analysis of this paper provides a theoretical rationale for the empirical evidence that even in times of severe market stress, such as a financial crisis, there may not be a marketwide funding freeze, but rather a market reallocation of funding liquidity from low-capital banks to high-capital banks. Thus, as documented by Perignon, Thesmar and Vuillemey (2018), some banks may actually *increase* their access to uninsured short-term funding, and thus dry-ups predict, but *do not cause*, future deterioration in bank performance.

Unlike the existing literature rationalizing government intervention during crises, taking funding dry-ups as given, this paper takes a step back and asks: what causes funding to dry up for some banks and not others, and what is the regulatory policy implication of this? The answer is difference in capital structure choices. Thus, regulatory directives for banks to hold more capital can help reduce the incidence of future funding dry-ups and thereby reduce taxpayer liability created by government intervention.

Related to this, the analysis also generates strategic complementarities in capital choices by banks, as indicated in the result below.

Proposition 5: *A higher capital level,* E_h^*, *chosen by high-capital banks leads to a higher capital level,* E_ℓ^*, *chosen by low-capital banks ceteris paribus.*

The intuition is as follows. When high-capital banks choose higher capital, they invest more in screening. This increases the probability that such a bank will detect a *G* loan chosen by a low-capital bank whose legacy loan it is considering buying, and it also leads to a higher expected price. This makes having a *G* loan in the crisis state more valuable for the low-capital bank, increasing the marginal value of screening and inducing it to choose higher capital. This result shows that there is "capital structure contagion", so banks can develop a "capital culture" in which the choice of higher capital levels by high-capital banks can generate a "rising tide" that lifts the capital levels of all banks.[14]

Of course, once the loan resale market clears, some low-capital banks will be unable to sell their legacy loans. But the banks that sell are those with good legacy loans, so the pool of banks with unsold loans now have worse assets on average than before interbank trading,

[14] Song and Thakor (forthcoming) develop a theory of bank culture in which culture is contagious in the sense that if some banks opt for safety-oriented cultures, so do others. That paper takes bank capital as exogenous and shows that higher capital induces a stronger bank preference for a safety-oriented culture.

government intervention to unfreeze *all* these banks will be more expensive than in the absence of interbank trading. Thus, it is also important that if the government does plan to intervene after interbank trading and buy assets at inflated prices (that are costly to taxpayers), this intervention should *not* be anticipated by the market. Otherwise, it will distort the loan resale market, and if the expected government subsidies for low-capital banks are high enough, the resale market will break down. These kinds of feedback loops are familiar from earlier research (e.g., Tirole (2012)).

V. WELFARE ANALYSIS

A. Welfare With Heterogeneous Beliefs

This section examines some of the welfare implications of the analysis. Because the model has heterogeneous beliefs, the standard welfare analysis has to be modified. We use Brunnermeier, Simsek and Xiong's (2014) approach. In that approach, there are heterogeneous belief and the social planner knows that one of the agents has the correct belief but does not know which agent. Their welfare criterion asserts a social choice to be "belief-neutral" efficient only if it is efficient under every reasonable belief.

Definition: A belief is reasonable if it is a convex combination of all agents' beliefs, as long as they are consistent with the commonly-agreed-upon aggregate statistics.

In our setting, the two beliefs are θ_1 and θ_2. The equilibrium derived in the previous section will be referred to as the "market equilibrium." One robust result that can be established right away is given below.

Proposition 6: *A social choice in which the low-capital bank keeps lower capital than in the market equilibrium and the high-capital bank keeps the same capital is belief-neutral inefficient relative to the market equilibrium.*

The intuition is related to the positive externality of bank capital.[15] If the optimistic bank (belief θ_1) keeps lower capital, it leads to a lower screening precision and hence a lower expected selling price for the legacy loan when $s=r$. This makes the pessimistic bank worse-off, using *its* belief to evaluate its expected payoff. Moreover, since the expected surplus from loan sales is shared between the high-capital and low-capital banks, one high-capital (belief θ_2) banks are also worse off, using *their* belief to evaluate their expected payoff. This is true for any convex combination of θ_1 and θ_2.

B. Pre-crisis Consolidation

One might wonder if encouraging the pessimistic banks to acquire optimistic banks at t=0 would help to avoid the inefficiency at t=1 that some low-capital banks with good legacy loans cannot finance their new asset at t=1. However, this solution runs into a basic problem. The pessimistic banks value the banks owned by the optimists *lower* than the optimists do. Thus, there will never be any trade at t=0.

The other impediment is adverse selection. Given the possible presence of the lemon banks, it is not feasible for the pessimists to raise enough financing to invest in their own loans and also purchase other banks at t=0.

C. Possible *Ex Post* Government Intervention

[15] See Proposition 5.

A scheme that the government could use to improve welfare would work as follows. After the interbank loan trading market clears, the government could buy the legacy loans of a *fraction* of the low-capital banks at $P_{\min}$, in exchange for a collective claim on the late t=2 cash flows of *all* the banks[16] whose legacy assets remain unsold after secondary market loan trading. The scheme can be announced at t=1 after interbank trading ends, but the identities of the banks whose loans are purchased are randomly chosen. This mechanism results in a *belief-neutral* welfare improvement.

Why? To see this, note that the government determines the fraction of banks whose loans it will purchase in order to *break even* based on its claim on *all* of the banks with unsold loans.[17] Since the number of banks against which the government has claims exceeds the number of banks from which loans are purchased, it is possible for the government to do this.[18] Moreover, since the purchase price is $P_{\min}$, it does not affect the equilibrium price at which trading occurs. The welfare improvement comes from the selling banks investing in their new assets at t=1.

VI. CONCLUSION

This paper has developed a theory in which the pre-crises capital choices of firms are driven by their beliefs about the probability of a future crisis, leading to heterogeneity in capital choices. When a crisis arrives, high-capital banks have better legacy assets and continue to have access to funding, but low-capital banks with poorer assets are frozen out of the market. The high-capital banks purchase legacy assets from low-capital banks and fund these purchases by *increasing* their short-term funding, whereas the low-capital banks that sell their legacy assets reduce their

[16] This includes banks whose legacy loans the government does not purchase.
[17] This is facilitated by the fact that it is the government implementing the scheme.
[18] So there is no cost to the taxpayers.

short-term funding. Thus, consistent with recent empirical evidence, there is a reallocation of market liquidity from low-capital to high-capital banks.

When a market is frozen by adverse selection, it appears to the central bank like a liquidity crisis that requires a marketwide liquidity injection. The analysis here shows how such a crisis may be rooted in low bank capital in *prior periods*, which generates the policy implication that a regulatory response that may be less costly to taxpayers would be to insist on higher bank capital in the pre-crisis (good) periods.

APPENDIX

Parametric Restrictions

Restriction R1

$$I < v\left[g + [1-g]\theta\right]X < 2I < X \tag{R-1}$$

Restriction R2

$\exists g^* \in (g,1)$ such that (R-2)

$$R = \frac{I}{g^*}$$

Define

$$f^* \equiv \frac{\bar{\eta} g}{\bar{\eta} g + [1-\bar{\eta}][1-g]} \tag{R-3}$$

$$f^{**} \equiv \frac{\bar{\eta} f^*}{\bar{\eta} f^* + [1-\bar{\eta}][1-f^*]} \tag{R-4}$$

Restriction R3

$$3I > X > \frac{2I+\varphi}{f^* f^{**}} \tag{R-5}$$

$$X[1-f^{**}] + \varphi > 2I\left[g^{-1}-1\right] \tag{R-6}$$

Combining (R-1), (R-5) and (R-6) gives us:

$$Max\left\{\frac{I}{v\left[g+[1-g]\theta\right]}, \frac{2I\left[g^{-1}-1\right]-4}{\left[1-f^*\right]}, \frac{2I+\varphi}{f^* f^{**}}\right\} < X < Min\left\{3I, \frac{2I}{v\left[g+[1-g]\theta\right]}\right\} \tag{R-7}$$

Restriction R4

$$\frac{I}{g} + R + \varphi < X < \frac{R-I-\varphi}{\left[1-g^2\right]} \tag{R-8}$$

and

$$I\left\{\frac{1-g^2}{g}+1\right\}+2\varphi<Rg^2 \tag{R-9}$$

Note that (R-9) ensures that $\frac{I}{g}+R+\varphi<\frac{R-I-\varphi}{\left[1-g^2\right]}$. It essentially requires that g is high enough. Given that φ is arbitrarily small, (R−9) clearly holds for g=1.

Restriction R5

$$\frac{[2g-1]}{[1-g]}>\frac{1}{2g} \tag{R-10}$$

Restriction R6

$\exists\eta^0\in\left(\underline{\eta},\overline{\eta}\right)$ such that

$$\frac{q\eta^0 gX}{\eta^0 g+\left[1-\eta^0\right]\left[1-g\right]}=2I \tag{R-11}$$

PROOFS

***Proof of Lemma 1*:** Since $R > I$, the bank wants to raise financing at t=1 to invest in the new asset. Moreover, given (R-1), it follows that investors will be willing to provide 2I in financing if they are sure that the bank has a G loan. When s=b, every bank has a G loan with probability 1. Thus, the bank will raise enough financing at t=1 to pay first-period depositors D and invest I in the new asset. Since the loan pays off X with probability 1 at t=2, the amount depositors need is D regardless of whether depositors are paid off at t=1 or t=2. ■

Proof of Lemma 2: When s=b, every bank is able to raise the external financing it needs at t=1. Interbank trading is inefficient since the purchasing bank incurs a cost of φ, which is avoided if the selling bank were to simply raise the necessary deposit financing at t=1. ■

Proof of Lemma 3: Follows from the arguments presented in the discussion following the lemma. The only additional detail is to prove that there cannot be a (partially) separating equilibrium in which all the banks with B loans raise financing at t=1 and the banks with G loans randomize, with some raising financing to pay off depositors at t=1 and others waiting until t=2 to repay. This is because in this case the banks that raise financing at t=1 should face a repayment obligation at t=2 that exceeds $D\left[f_\ell\right]^{-1}$, whereas those that wait until t=2 to repay first-period depositors should face a repayment obligation of D. Thus, the banks with G loans will not be indifferent across the two alternatives. ■

***Proof of Proposition 1*:** Assume $f_\ell < g^*$. Suppose a bank that knows at t=1 that it has a G loan attempts to raise I to purchase the new asset. Its repayment obligation (assuming it can raise the financing) will be $I\left[f_\ell\right]^{-1}$. If it also pays off its first-period depositors, it will need to borrow D in additional deposits with a repayment obligation of $D\left[f_\ell\right]^{-1}$ at t=2. Thus, a bank will view its expected payoff from raising D+I at t=1 as:

$$\left\{X - D\left[f_\ell\right]^{-1} - I\left[f_\ell\right]^{-1}\right\} + R \tag{A-1}$$

If it chooses to not finance the new asset and only pay off first-period depositors, its expected payoff will be:

$$X - D\left[f_\ell\right]^{-1} \tag{A-2}$$

Thus, it will prefer to *not* raise financing for the new asset if the expression in (A-2) exceeds that in (A-1), or if

$$R - I\left[f_\ell\right]^{-1} < 0 \tag{A-3}$$

which is true since $f_\ell < g^*$ (see (R-2)).

Now the bank that knows it has a G loan is indifferent between raising financing at t=1 to pay off first-period depositors and waiting until t=2 to pay them off, if the terms of the deposit contract are as in Lemma 3, and any financing raised at t=1 involves all banks (see the proof of Lemma 3). But if all banks decide to raise financing at t=1, then the banks with B loans will strictly prefer to consume D at t=1 and take advantage of the terms of the deposit contract that permit a repayment of $D\left[f_\ell\right]^{-1}$ at t=2 that they know they will not make. However, this strategy is not optimal for the banks with G loans since their expected payoff from doing so is:

$$\left[X - \frac{2D}{f_\ell}\right] + D \tag{A-4}$$

where it is assumed that all banks are raising D at t=1 (so the promised repayment will be $D\left[f_\ell\right]^{-1}$ on these deposits, which is added to the repayment of $D\left[f_\ell\right]^{-1}$ on the first-period deposits). If these banks use the D raised at t=1 to pay off first-period deposits, then their expected payoff at t=2 is:

$$\left[X - \frac{D}{f_\ell}\right] \tag{A-5}$$

It is clear that the expression in (A-5) exceeds that in (A-4). Hence, the banks with G loans will always pay off first-period depositors at t=1 if they raise D at t=1. This means that under the terms of the deposit contract in Lemma 3, they are indifferent between paying off first-period depositors at t=1 and paying them off at t=2. Thus, it is a sequential equilibrium in which no bank raises financing at t=1, supported by the belief that any bank that attempts to raise financing at t=1 is a bank with a B loan. This passes the universal divinity refinement since the set of beliefs (and hence the set of repayment obligations at t=2) about the type of bank attempting to raise financing at t=1 that would induce the bank with the G loan to raise financing is strictly nested within the corresponding set for the bank with the B loan. Hence, depositors are unwilling to provide any financing to low-capital banks at t=1. ■

Proof of Lemma 4: Since a high-capital bank screens with precision η_h, when $s=r$, the probability that the bank has a G loan is:

$$\Pr\left(G \mid \text{high capital}\right) = f_h = \frac{\eta_h g}{\eta_h g + [1-\eta_h][1-g]} \tag{A-6}$$

And if another high-capital bank screens the loan of a high-capital bank at t=1 to purchase it, then conditional on its signal saying that the loan is G,

$$\Pr(G) = \hat{\hat{f}}_h^r = \frac{\eta_h f_h}{\eta_h f_h + [1-\eta_h][1-f_h]} \tag{A-7}$$

The maximum price the buyer would be willing to pay for the loan is:

$$P_{\max}^h = \hat{\hat{f}}_h^r X - \varphi \tag{A-8}$$

Now if the potential seller with a G loan does not sell the loan, its expected payoff at t=2 is

$$\left[X - [D_h + I][f_h]^{-1}\right] + R \tag{A-9}$$

where D_h is the debt level of the high-capital bank, assuming it raises $D_h + I$ in financing and purchases the new asset. If it sells its legacy loan at a price P, its expected payoff is:

$$P + R - \left(I + D_h\right) \tag{A-10}$$

Let $P_{\min}$ be the minimum price the seller will accept. It is the value of P at which (A-9) and (A-10) are equal. Thus,

$$P_{\min}^h = X - [D_h + I]\left[f_h^{-1} - 1\right] \tag{A-11}$$

For there to be no trade, we need

$$P_{\min}^h > P_{\max}^h \tag{A-12}$$

where $P_{\max}^{h}$ is given by (A-8). That is,

$$X\left[1-\hat{\hat{f}}_h^{\,r}\right]+\varphi>\left[D_h+I\right]\left[f_h^{-1}-1\right] \tag{A-13}$$

It is clear that this condition is satisfied given (R-6), since $\hat{\hat{f}}_h^{\,r}\leq f^{**}$ and $f_h^{-1}<g^{-1}$. Thus, there is no trade. ■

Proof of Lemma 5: The proof follows from the fact that the maximum price a low-capital bank can pay for the legacy loan of another low-capital bank

$$P_{\max}^{\ell}=\hat{\hat{f}}_{\ell}^{\,r}X-\varphi \tag{A-14}$$

where

$$\hat{\hat{f}}_{\ell}^{\,r}=\frac{\eta_{\ell}f_{\ell}}{\eta_{\ell}f_{\ell}+\left[1-\eta_{\ell}\right]\left[1-f_{\ell}\right]} \tag{A-15}$$

Since $\eta_{\ell}<\eta_h$, it follows that $P_{\max}^{\ell}<P_{\max}^{h}=f_h\hat{f}_hX-\varphi$. ■

Proof of Lemma 6: It is clear from (8) and (9) that $P_{\max}^{1}>P_{\max}^{2}$. Note that given (R-5), it must be the case that $P_{\max}=P_{\max}^{2}$. Thus, we need to show the existence of gains from trade for $P_{\max}=P_{\max}^{2}$. The proof first requires showing that $P_{\max}>P_{\min}$. So we need

$$f_h\hat{f}_h^{\,r}X-\varphi>\left[X-D_h\left\{f_{\ell}\right\}^{-1}\right]D_{\ell}+I-R \tag{A-16}$$

which, upon rearranging, yields:

$$R>X\left[1-f_h\hat{f}_h^{\,r}\right]+\varphi+I-D_{\ell}\left[f_{\ell}^{-1}-1\right] \tag{A-17}$$

Since $f_h\geq g$, $\hat{f}_h^{\,r}\geq g$, (R-8) is sufficient for (A-17) to hold.

Next, it will be verified that

$$P_{\min}-\varphi>D_{\ell}+I. \tag{A-18}$$

Substituting for $P_{\min}$ from (7), we need

$$X-D_{\ell}\left[f_{\ell}\right]^{-1}+D_{\ell}+I-R>D_{\ell}+I \tag{A-19}$$

Since $f_\ell \geq g$, it follows from (R-8) that(A-18) holds. ■

Proof of Lemma 7: The first-order condition on η_ℓ is:

$$FOC(\eta_\ell) \equiv [1-\theta_1]\left\{\begin{array}{l}[\partial\tau/\partial\eta_\ell]\left[\bar{P}-D_\ell+R-I-f_\ell\left[X-D_\ell\{f_\ell\}^{-1}\right]\right] \\ +[1-\tau][\partial f_\ell/\partial\eta_\ell]\left[X-D_\ell\{f_\ell\}^{-1}\right]\end{array}\right\}-C'=0 \quad \text{(A-20)}$$

where

$$\begin{aligned}\partial\tau/\partial\eta_\ell &= [\partial f_\ell/\partial\eta_\ell]\eta_h-[\partial f_\ell/\partial\eta_\ell][1-\eta_h] \\ &= [\partial f_\ell/\partial\eta_\ell][2f_h-1]>0 \text{ since } \eta_h>0.5\end{aligned} \quad \text{(A-21)}$$

and using (4):

$$\partial f_\ell/\partial\eta_\ell = \frac{g[1-g]}{\{\eta_\ell g+[1-\eta_\ell][1-g]\}^2}>0 \quad \text{(A-22)}$$

Further,

$$\partial^2\tau/\partial\eta_\ell^2 = [\partial\eta_h-1]\left[\partial^2 f_\ell/\partial\eta_\ell^2\right] \quad \text{(A-23)}$$

$$\begin{aligned}\partial^2 f_\ell/\partial\eta_\ell^2 &= \frac{-2g[1-g][2g-1]}{\{\eta_\ell g+[1-\eta_\ell][1-g]\}^3} \\ &<0 \text{ since } g\geq 0.5\end{aligned} \quad \text{(A-24)}$$

Using (A-20), the second-order condition for η_ℓ is:

$$SOC(\eta_\ell)\equiv\partial^2\Omega/d\eta_\ell^2 = [1-\theta_1]\left\{\begin{array}{l}[2\eta_h-1]\left[\partial^2 f_\ell/\partial\eta_\ell^2\right]\{P-D_\ell+R-I-f_\ell X-D\} \\ -[2\eta_h-1][\partial f_\ell/\partial\eta_\ell]^2\left[X-D_\ell\{f_\ell\}^{-1}\right] \\ +[1-\tau]\left[\partial^2 f_\ell/\partial\eta_\ell^2\right]\left[X-D_\ell\{f_\ell\}^{-1}\right]\end{array}\right\}-C''<0 \quad \text{(A-25)}$$

Since $\partial^2 f_\ell/\partial\eta_\ell^2<0$, (A-25) clearly holds.

Finally, we prove $d\eta_\ell^*/dE>0$.

Totally differentiating the first-order condition (A-20) and substituting $D_\ell = I-E$, we have:

$$\left[SOC(\eta_\ell)\right]\left[d\eta_\ell^*/dE_\ell\right]+[1-\theta_1]\left\{[1-\tau][\partial f_\ell/\partial\eta_\ell][f_\ell]^{-1}\right\}=0 \quad \text{(A-26)}$$

Thus,

$$\frac{d\eta_\ell^*}{dE_\ell} = \frac{-[1-\theta_1][1-\tau][\partial f_\ell / \partial \eta_\ell][f_\ell]^{-1}}{SOC(\eta_\ell)} > 0 \tag{A-27}$$

■

Proof of Lemma 8: First, it will be proved that $\partial P_{\max} / \partial E_\ell > 0$. Using $P_{\max} = P_{\max}^2$ from (8), we have:

$$\partial P_{\max} / \partial E_\ell = f_h X \left\{ \frac{\partial \hat{f}_h^r}{\partial \eta_\ell} \bullet \frac{\partial \eta_\ell}{dE_\ell} \right\} > 0 \tag{A-28}$$

since $\partial \eta_\ell / dE_\ell > 0$ by Lemma 7, and

$$\partial \hat{f}_h^r / \partial f_\ell = \frac{\eta_h [1-\eta_h]}{\{\eta_h f_\ell + [1-\eta_h][1-f_\ell]\}^2} > 0 \tag{A-29}$$

Moreover,

$$\frac{\partial f_\ell}{\partial \eta_\ell} = \frac{g[1-g]}{\{\eta_\ell g + [1-\eta_\ell][1-g]\}^2} > 0 \tag{A-30}$$

Next, use $D_\ell = I - E_\ell$ to write

$$\partial P_{\min} / \partial E_\ell = [f_\ell^{-1} - 1] + [I-E] f_\ell^{-2} [\partial f_\ell / \partial \eta_\ell] \left[\frac{d\eta_\ell}{dE_\ell} \right] > 0 \tag{A-31}$$

since $\partial f_\ell / \partial \eta_\ell > 0$, $d\eta_\ell / dE_\ell > 0$.

Next, it will be proved that $\overline{P}$ is concave in E_ℓ. It is easiest to do this by proving that $P_{\min}$ and $P_{\max}$ are individually concave in E_ℓ.

Now,

$$\begin{aligned} \partial^2 P_{\min} / \partial E_\ell^2 = & -2 f_\ell^{-2} \left[\frac{\partial f_\ell}{\partial \eta_\ell} \bullet \frac{d\eta_\ell}{dE_\ell} \right] - 2[1-E_\ell] f_\ell^{-3} \left[\frac{\partial f_\ell}{\partial \eta_\ell} \bullet \frac{d\eta_\ell}{dE_\ell} \right]^2 \\ & + [I - E_\ell] f_\ell^{-2} \left\{ \left[\frac{\partial^2 f_\ell}{\partial \eta_\ell^2} \right] \left(\frac{d\eta_\ell}{dE_\ell} \right)^2 + \left[\frac{\partial f_\ell}{\partial \eta_\ell} \right] \left[\frac{d^2 \eta_\ell}{dE_\ell^2} \right] \right\} \end{aligned} \tag{A-32}$$

Now, $\partial f_\ell / \partial \eta_\ell > 0$, $d\eta_\ell / dE_\ell > 0$, $\partial^2 f_\ell / \partial^2 \eta_\ell < 0$. So, what is the sign of $\partial^2 \eta_\ell / \partial E_\ell^2$? Using (A-27), we can write:

$$\partial^2\eta_\ell / \partial E_\ell^2 = \left\{ \frac{\begin{matrix} SOC(\eta_\ell)[1-\theta_1][1-\tau]\left\{ f_\ell^{-2}[\partial f_\ell / \partial \eta_\ell]^2 - f_\ell^{-1}\left[\partial^2 f_\ell / \partial \eta_\ell^2\right]\right\} \\ \Delta_1\{\partial SOC(\eta_\ell)/\partial\eta_\ell\}[d\eta_\ell / dE_\ell] \end{matrix}}{\left[SOC(\eta_\ell)\right]^2} \right\} \tag{A-33}$$

Where

$$A_1 \equiv -[1-\theta_1][1-\tau][\partial f_\ell / \partial \eta_\ell][f_\ell]^{-1} < 0 \tag{A-34}$$

Clearly, $\partial SOC(\eta_\ell)/\partial\eta_\ell < 0$ for $C''' > 0$ sufficiently large. Thus, $\partial^2\eta_\ell / \partial E_\ell^2 < 0.$ With this, it is clear from (A-32) that $\partial^2 P_{\min} / \partial E_\ell^2 < 0.$ Now, consider $P_{\max}$. Using (8), we can write:

$$\partial P_{\max} / \partial E_\ell = \left[\partial \hat{f}_h^r / \partial \eta_\ell\right] \bullet [d\eta_\ell / dE_\ell] \tag{A-35}$$

We have already shown that $d\eta_\ell / dE_\ell > 0$ and $d^2\eta_\ell / dE_\ell^2 < 0.$ So,

$$\underbrace{\partial^2 \hat{f}_h^r / \partial \eta_\ell} = \frac{[\partial f_\ell / \partial \eta_\ell]\eta_h[1-\eta_h]}{\left[\eta_h f_\ell + [1-\eta_h][1-f_\ell]\right]^2} > 0. \tag{A-36}$$

And

$$\partial^2 f^r / \partial \eta_\ell^2 = \frac{(A_2)^2 \eta_h[1-\eta_h]\left[\partial^2 f_\ell / \partial\eta_\ell^2\right] - 2A_2[\partial f_\ell / \partial \eta_\ell]^2[\partial\eta_h - 1]}{A_2^4} \tag{A-37}$$

where $A_2 \equiv \eta_h f_\ell + [1-\eta_h][1-f_\ell].$ (A-38)

Since $\partial^2 f_\ell / \partial\eta_\ell^2 < 0,$ it follows from (A-37) that $\partial^2 \hat{f}^r / \partial\eta_\ell^2 < 0.$ Since $\partial^2 \hat{f}_h^r / \partial\eta_\ell > 0$ above, it follows from (A-35) that

$$\partial P_{\max} / \partial E_\ell > 0. \tag{A-39}$$

Further,

$$\partial^2 P_{\max} / \partial E_\ell^2 = \left[\partial \hat{f}_h^r / \partial\eta_\ell\right]\left[\partial^2\eta_\ell / \partial E_\ell^2\right] + [d\eta_\ell / dE_\ell]\left[\partial^2 \hat{f}_h^r / \partial\eta_\ell^2\right] \tag{A-40}$$

Given $\partial\hat{f}_h^r / \partial\eta_\ell > 0,$ $d^2\eta_\ell / dE_\ell^2 < 0,$ $d\eta_\ell / dE_\ell > 0,$ and $\partial^2 \hat{f}_h^r / \partial\eta_\ell^2 < 0,$ it follows from (A-40) that $\partial^2 P_{\max} / \partial E_\ell^2 < 0.$ ■

Proof of Proposition 2: The first-order condition for is:

$$\theta_1 + [1-\theta_1]\left[\tau\left\{d\bar{P} / dE_\ell + 1\right\} + \{1-\tau\} f_\ell[f_\ell]^{-1}\right] - 1 - \psi' + [d\Omega / d\eta]\left[d\eta_\ell^* / dE_\ell\right] = 0 \tag{A-41}$$

where Ω is defined in (11).

Using the Envelope Theorem, we have:

$$\theta_1 + [1-\theta_1]\tau\left[d\bar{P} / dE_\ell\right] - 1 - \psi' = 0 \tag{A-42}$$

The second-order condition is:

$$SOC(E) = -\Psi''[1-\theta_1]\tau\left[d^2\bar{P} / dE_\ell^2\right] + \left[d^2\Omega / d\eta^2\right]\left[d\eta_\ell^* / dE_\ell\right] + \left[d\Omega / d\eta_\ell\right]\left[d\eta_\ell^* / dE_\ell\right] < 0 \tag{A-43}$$

Again using the Envelope Theorem, we have:

$$SOC(E) = -\Psi''[1-\theta_1]\tau\left[d^2\bar{P} / dE_\ell^2\right] + SOC(\eta_\ell)\left[d\eta_\ell^* / dE_\ell\right] \tag{A-44}$$

Since $d^2\bar{P} / dE_\ell^2 \le 0,$ $SOC(\eta_\ell) < 0$ and (Lemma 7), we see that $SOC(E) < 0.$ Finally, we prove that $dE_\ell^* / d\theta_1 < 0.$ Totally differentiating the FOC (A-41) gives us:

$$1 - \tau\left[d\bar{P} / dE_\ell\right] + SOC(E)\left(dE_\ell^* / d\theta_1\right) = 0 \tag{A-45}$$

Thus,

$$\frac{dE_\ell^*}{d\theta_1} = \frac{\tau\left[d\bar{P} / dE_\ell\right] - 1}{SOC(E)} \tag{A-46}$$

Now, from (A-42), we know

$$\left[\tau\left[d\bar{P} / dE_\ell\right] - 1\right][1-\theta_1] = \Psi'$$

which means

$$\tau\left[d\bar{P} / dE_\ell\right] - 1 = \frac{\Psi'}{[1-\theta_1]} > 0 \tag{A-47}$$

Thus, from (A-46), it follows that

$$dE_\ell^* / d\theta_1 < 0.$$

■

Proof of Lemma 9: Now

$$\partial^2\left[f_h \hat{f}_h^r\right] / \partial\eta_h^2 = \left[\partial^2 f_h / \partial\eta_h^2\right]\hat{f}_h^r + \left[\partial^2 \hat{f}_h^r / \partial\eta_h^2\right] f_h + 2\left[\frac{\partial f_h}{\partial\eta_h}\right]\left[\frac{\partial \hat{f}_h^r}{\partial\eta_h}\right] \tag{A-48}$$

$$= \frac{-g[1-g]f_\ell\left\{\frac{2[2g-1]\eta_h}{\eta_h g+[1-\eta_h][1-g]}-\frac{[1-f_\ell]}{\eta_h f_\ell+[1-\eta_h][1-f_\ell]}\right\}}{\{\eta_h g+[1-\eta_h][1-g]\}^2\{\eta_h f_\ell+[1-\eta_h][1-f_\ell]\}} \tag{A-49}$$

$$= \frac{-f_\ell[1-f_\ell]g\left\{\frac{2[2f_\ell-1]g}{\eta_h f_\ell+[1-\eta_h][1-f_\ell]}-\frac{[1-g]}{\eta_h g+[1-\eta_h][1-g]}\right\}}{\{\eta_h f_\ell+[1-\eta_h][1-f_\ell]\}^2\{\eta_h g+[1-\eta_h][1-g]\}}$$

The above will be negative if

$$\frac{2[2g-1]\eta_h}{\eta_h g+[1-\eta_h][1-g]} > \frac{[1-f_\ell]}{\eta_h f_\ell+[1-\eta_h][1-f_\ell]} \tag{A-50}$$

and

$$\frac{2[2f_\ell-1]\eta_h}{\eta_h f_\ell+[1-\eta_h][1-f_\ell]} > \frac{[1-g]}{\eta_h g+[1-\eta_h][1-g]} \tag{A-51}$$

Note that the left-hand side (LHS of (A-51) is increasing in *g*, the right-hand side (RHS) of (A-50) is decreasing in f_ℓ (note $g,\eta_h > 0.5$), and f_ℓ is increasing in *g*. Since $f_\ell > g$, the LHS of (A-50) is less than the LHS of (A-51). So as *g* increases, the RHS of (A-50) and (A-51) decrease and the LHS of (A-50) and (A-51) increase. Moreover, if (A-51) holds, so will (A-50). So for (A-51) to hold, we need:

$$\begin{aligned}&\left[2[2f_\ell-1]\eta_h\right]\{\eta_h g+[1-\eta_h][1-g]\}\\ &>[1-g]\{\eta_h f_\ell+[1-\eta_h][1-f_\ell]\}\end{aligned} \tag{A-52}$$

Since $\eta_h+[1-\eta_h][1-g] > \eta_h\eta_\ell g+[1-\eta_\ell][1-\eta_h][1-g]$, what is needed for (A-52) to hold is:

$$2[2f_\ell-1]\eta_h > \frac{[1-g]}{\eta_\ell g+[1-\eta_\ell][1-g]} \tag{A-53}$$

which is equivalent to showing that

$$\begin{aligned}g[1-g]^{-1} &> \frac{2\eta_h[1-\eta_\ell]+1}{2\eta_\ell\eta_h}\\ &= \frac{[1-\eta_\ell]}{\eta_\ell}+\frac{1}{2\eta_\ell\eta_h}\end{aligned} \tag{A-54}$$

Since the RHS of (A-54) is decreasing in η_ℓ and η_h, it is clear that (9) is sufficient for (A-54) to hold. ■

Proof of Lemma 10: The first-order condition (FOC) for η_h^* is (using (14) and noting that the actual unobserved choice of η_h does not affect $\bar{\bar{P}}$ or P_R as these depend only on the equilibrium choices the bank is believed to have made):

$$\begin{aligned}&=\left[1-\theta_2\right]\left\{\begin{array}{l}\tau\left\{\left[\partial f_h/\partial\eta_h\right]\left[\hat{f}_h^r\right]+\left[\partial\hat{f}_h^r/\partial\eta_h\right]\left[f_h\right]\right\}H_1\\+\left[1-\tau\right]\left[\partial f_h/\partial\eta_h\right]H_2+\left[\partial\tau/\partial\eta_h\right]\left[f_h\hat{f}_h^r H_1-f_h H_2\right]\end{array}\right\}-C'\\&=0\end{aligned} \tag{A-55}$$

where

$$H_1\equiv 2X-\varphi-P_R-\left[D_0(h)+I\right]\left[f_h\right]^{-1} \tag{A-56}$$

$$H_2\equiv X-\left[D_0(h)+I\right]\left[f_h\right]^{-1} \tag{A-57}$$

The second-order condition is:

$$\begin{aligned}&\partial^2\Omega_h/\partial\eta_h^2\equiv SOC\left(\eta_h\right)\\&=\left[1-\theta_2\right]\left\{\tau H_1H_3+\left[1-\tau\right]\left[\partial^2 f_h/\partial\eta_h^2\right]H_2+\left[f_h\hat{f}_h^r H_1-f_hH_2\right]\left[\partial^2\tau/\partial\eta_h^2\right]\right\}-C''\\&<0\end{aligned} \tag{A-58}$$

where

$$\begin{aligned}H_3\equiv&\left[\partial^2 f_h/\partial\eta_h^2\right]\left[\hat{f}_h^r\right]+2\left[\partial f_h/\partial\eta_h\right]\left[\partial\hat{f}_h^r/\partial\eta_h\right]\\&+\left[\partial^2\hat{f}_h^r/\partial\eta_h^2\right]\left[f_h\right]\\&<0 \text{ by Lemma 9,}\end{aligned} \tag{A-59}$$

and $\partial^2\tau/\partial\eta_h^2<0$. Thus, since $\partial^2 f_h/\partial\eta_h^2<0$, $H_1>0$, $H_2>0$, we see that (A-58) holds.

Finally, we prove $d\eta_h^*/dE_h>0$. Totally differentiating the FOC in (A-55):

$$\begin{aligned}&\left[1-\theta_2\right]\left\{\tau H_4\left[f_h\right]^{-1}+\left[-\tau\right]\left[\partial f_h/\partial\eta_h\right]\left[f_h\right]^{-1}\right\}\\&+SOC\left(\eta_h\right)\bullet\left[d\eta_h^*/dE_h\right]=0\end{aligned} \tag{A-60}$$

where

$$H_4\equiv\left[\partial f_h/\partial\eta_h\right]\left[\hat{f}_h^r\right]+\left[\partial\hat{f}_h^r/\partial\eta_h\right]\left[f_h\right] \tag{A-61}$$

This yields:

$$\begin{aligned}d\eta_h^*/dE_h=&\frac{-\left[1-\theta_2\right]\left\{\tau H_4\left[f_h\right]^{-1}+\left[1-\tau\right]\left[\partial f_h/\partial\eta_h\right]\left[f_h\right]^{-1}\right\}}{SOC\left(\eta_h\right)}\\&>0\text{ since }SOC\left(\eta_h\right)<0\end{aligned} \tag{A-62}$$

■

Proof of Proposition 3: The FOC for *E*, using (14) and the Envelope Theorem is:

$$\begin{aligned}\partial\Omega_h / \partial E_h &\equiv FOC(E_h) \\ &= \theta_2 + (1-\theta_2)\begin{Bmatrix}\tau\,\hat{f}_h^r + [1-\tau] \\ -\tau\, f_h\,\hat{f}_h^r\left[\partial R_P / \partial E_h\right]\end{Bmatrix} - 1 - \Psi' \\ &= 0\end{aligned} \tag{A-63}$$

where R_P is given in (15) and $\partial R_P / \partial E_h = -z_2 z_3^{-1} < 0.$

Given that $\partial R_P / \partial E_h = -z_2 z_3^{-1} < 0,$ we have:

$$\tau\, f_h\,\hat{f}_h^r\left[\partial R_P / \partial E_h\right] = \tau\, f_h\,\hat{f}_h^r\left[\frac{-1}{f_h^2} - \frac{1}{f_h\,\hat{f}_h^r} + \frac{2}{f_h}\right] \tag{A-64}$$

$$= \tau\left[-\frac{\hat{f}_h^r}{f_h} - 1 + 2\,\hat{f}_h^r\right]$$

where f_h is given in (4) and $\hat{f}_h^r$ is given in (5). Thus,

$$\tau\,\hat{f}_h^r - \tau\, f_h\,\hat{f}_h^r\left[\partial R_P / \partial E_h\right] = \left\{\frac{\hat{f}_h^r}{f_h} + 1 - \hat{f}_h^r\right\} \tag{A-65}$$

Now $\partial\left[\hat{f}_h^r / f_h\right] / \partial\eta_h,\ \partial\hat{f}_h^r / \partial\eta_h > 0$ and $d\eta_h^* / dE_h > 0.$ Thus, defining

$$H_5 \equiv \tau\,\hat{f}_h^r q - \tau\, f_h\,\hat{f}_h^r\left[\partial R_P / \partial E_h\right] \tag{A-66}$$

we know that

$$\partial H_5 / \partial E_h < 0 \tag{A-67}$$

Now write the second-order condition for E_h :

$$\begin{aligned}\partial^2\Omega_h / \partial E_h^2 &\equiv SOC(E_h) \\ &= [1-\theta_2]\left[\partial H_5 / \partial E_H\right] - \Psi'' + SOC(\eta_h)\left[d\eta_h^* / dE_h\right] + FOC(\eta_h)\left[d^2\eta_h^* / dE_h^2\right] \\ &< 0\end{aligned} \tag{A-68}$$

Since $SOC(\eta_h) = 0$ and $SOC(\eta_h) < 0,$ we can use (A-67) to confirm that $SOC(E_h) < 0.$ Finally, we prove that $dE_h^* / d\theta_2 < 0.$

Totally differentiating the FOC in (A-63):

$$\begin{aligned}\frac{dFOC(E_h)}{d\theta_2} &= 1 - \left[H_5 + [1-\tau]\right] + SOC(E_h)\frac{dE_h^*}{d\theta_2} \\ &= 0\end{aligned} \tag{A-69}$$

This yields $\frac{dE_h^*}{d\theta_2} = \frac{H_5 - \tau}{SOC(E_h)}$ (A-70)

where

$$H_5 - \tau = \tau \hat{f}_h^r \left[f_h^{-1} - 1 \right] = 0$$

Thus, since $SOC(E_h) < 0$,

$$\frac{dE_h^*}{d\theta_2} < 0$$ ■

Proof of Proposition 4: We ask: when is

$$\Omega_h^* > \Omega_\ell^* \tag{A-71}$$

where Ω_h^* and Ω_ℓ^* are given in (22) and (25) respectively. With a little algebra, we see that (A-71) holds if:

$$\begin{aligned} &\theta\left\{D_0(\ell) - D_0(h)\right\} + (1-\theta)\begin{Bmatrix} [R-I][1-\tau] + \tau I\left[1 - \hat{f}_h^r\right] \\ +\tau\begin{bmatrix} f_h \hat{f}_h^r \{2X - \varphi - R_P\} \\ -D_0(h)\hat{f}_h^r - \overline{\overline{P}} + D_0(\ell) \end{bmatrix} \\ +[1-\tau]\begin{bmatrix} f_h X - D_0(h) \\ -f_\ell X + D_0(\ell) \end{bmatrix} \end{Bmatrix} \\ &> E_h^* + \Psi\left(E_h^*\right) - \left[E_\ell^* + \Psi\left(E_\ell^*\right)\right] - \left[C\left(\eta_h^*\right) - C\left(\eta_\ell^*\right)\right] \end{aligned} \tag{A-72}$$

Now define

$$\begin{aligned} \phi_1 &\equiv \tau\left[D_0(\ell) - \hat{f}_h^r D_0(h)\right] + [1-\tau]\left[D_0(\ell) - D_0(h)\right] \\ &> D_0(\ell) - D_0(h) \end{aligned} \tag{A-73}$$

Thus, a sufficient condition for (A-72) to hold can replace ϕ by $D_0(\ell) - D_0(h)$ on the LHS of (A-72).

That is, we need

$$\begin{aligned} &[1-\theta]\left[D_0(\ell) - D_0(h)\right] + \theta\left[D_0(\ell) - D_0(h)\right] + [1-\theta]\begin{Bmatrix} [R-I][1-\tau] + \tau I\left[1 - \hat{f}_h^r\right] \\ +\tau\left[f_h \hat{f}_h^r\left[2X - \varphi - R_p\right] - \overline{P}\right] \\ +[1-\tau]\left[f_h - f_\ell\right]X \end{Bmatrix} \\ &> \phi_2 \end{aligned} \tag{A-74}$$

where $$\begin{aligned} \phi_2 &\equiv E_h^* + \psi\left(E_h^*\right) - \left[E_\ell^* + \psi\left(E_\ell^*\right)\right] - \left[C\left(\eta_h^*\right) - C\left(\eta_\ell^*\right)\right] \\ &> 0 \end{aligned} \tag{A-75}$$

Now substituting for R_p from (18), we see that

$$f_h \hat{f}_h^r\left[2X-\varphi-R_p\right]-\bar{P}=2 f_h \hat{f}_h^r X-\varphi-\bar{P}+\left[z_2 z_3^{-1}\right]\left[f_h X-D_0(h)-I\right]$$

Thus, $f_h \hat{f}_h^r\left[2X-\varphi-R_p\right]-\bar{\bar{P}}$

$$\begin{aligned}
&=2 f_h \hat{f}_h^r X-\varphi-2\bar{\bar{P}}+f_h \hat{f}_h^r\left[z_2 z_3^{-1}\right]\left[f_h X-D_0(h)-I\right]\\
&>2\left[f_h \hat{f}_h^r q^2 X-\frac{\varphi}{2}-P\right] \text{ (using (9))}\\
&>2\left[P_{\max}-\bar{\bar{P}}\right]\\
&>0
\end{aligned}$$

Thus, (A-74) becomes

$$D_0(\ell)-D_0(h)+[1-\theta]\phi_3>\phi_2 \tag{A-76}$$

where

$$\phi_3 \equiv [R-I][1-\tau]+\tau I\left[1-\hat{f}_h^r\right]+\tau\left[f_h \hat{f}_h^r\left\{2X-\varphi-R_p\right\}-\bar{\bar{P}}\right]+[1-\tau]\left[f_h-f_\ell\right]X \tag{A-77}$$

Thus, the LHS of (A-76) is strictly decreasing in θ and the inequality in (A-75) will only hold for θ low enough. That is, $\exists\theta^*$ such that the bank will choose high capital if $\theta<\theta^*$ and low capital if $\theta>\theta^*$. The last part is to prove that if θ is low enough, then $\eta_h>\eta^0$. This result follows from the fact that E_h^* is decreasing in θ (or θ_2) and η_h^* is increasing in E_h^*. ■

Proof of Proposition 5: Totally differentiating the first-order condition (A-41) yields:

$$[1-\theta_1]\left\{\frac{\partial\tau}{\partial E_h^*}\right\}\left\{\frac{d\bar{P}}{\partial E_\ell^*}\right\}+SOC\left(E_\ell\right)\left[dE_\ell^*/E_h^*\right]=0 \tag{A-78}$$

which gives

$$\frac{dE_\ell^*}{dE_h^*}=\frac{-\left[1-\theta_1\right]\left[\partial\tau/\partial E_h^*\right]\left[d\bar{P}/dE_\ell^*\right]}{SOC\left(E_\ell\right)} \tag{A-79}$$

Since $\partial\tau/\partial E_h^*>0$, $d\bar{P}/dE_\ell^*>0$, and $SOC\left(E_\ell\right)<0$, it follows that $dE_\ell^*/dE_h^*>0$. ■

Proof of Proposition 6: Obvious from the discussion in the text. ■

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